Name: Key

Instructor/College:

Section:

Score: /200

Department of Electrical & Computer Engineering

Electrical Engineering Advancement Exam I

FALL SEMESTER 2000

CLOSED BOOK

2 HOUR TIME LIMIT

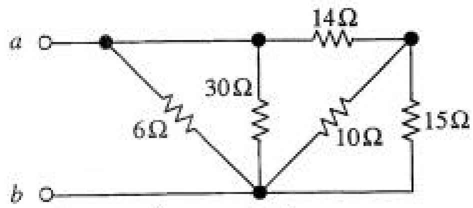
CALCULATORS ARE ALLOWED

There are ten (10) problems; please look over your exam to make sure you have ten (10) different problems. DO ANY EIGHT (8) PROBLEMS! Draw a large X through the two (2) problems that you do not want to be graded. If you do not indicate which problems you want to leave out, the first eight (8) problems will be graded.

Do all work for each problem only on the page(s) supplied for that problem. DO NOT, for instance, continue Problem 3 on the back of Problem 2. Extra blank paper will be supplied if needed. If extra paper is used, show the additional work for each problem on a separate sheet and staple the extra sheet(s) to the appropriate problem(s).

1. Determine the value of the equivalent element connected between terminals a and b.

a) R_{eq} = ___4___ Ω

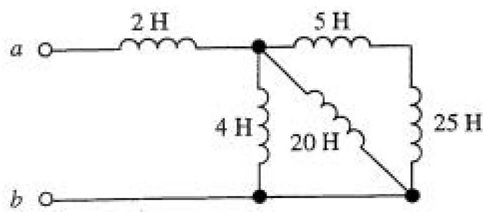


$R_{p_1} = \dfrac{1}{\frac{1}{10}+\frac{1}{15}} = 6\Omega$

$R_{s_1} = 14+6 = 20\Omega$

$R_{p2} = \dfrac{1}{\frac{1}{6}+\frac{1}{30}+\frac{1}{20}} = 4\Omega$

b) L_{eq} = ___5___ H

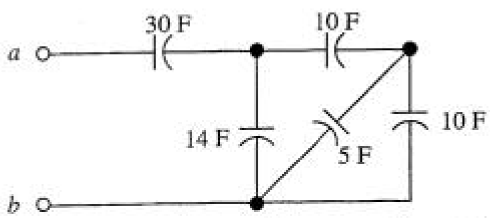


$L_{s_1} = 5+25 = 30H$

$L_{p_1} = \dfrac{1}{\frac{1}{4}+\frac{1}{20}+\frac{1}{30}} = 3\ H$

$L_{s2} = 2+3 = 5\ H$

c) C_{eq} = ___12___ F



$C_{p_1} = 5+10 = 15\ F$

$C_{s_1} = \dfrac{1}{\frac{1}{10}+\frac{1}{15}} = 6F$

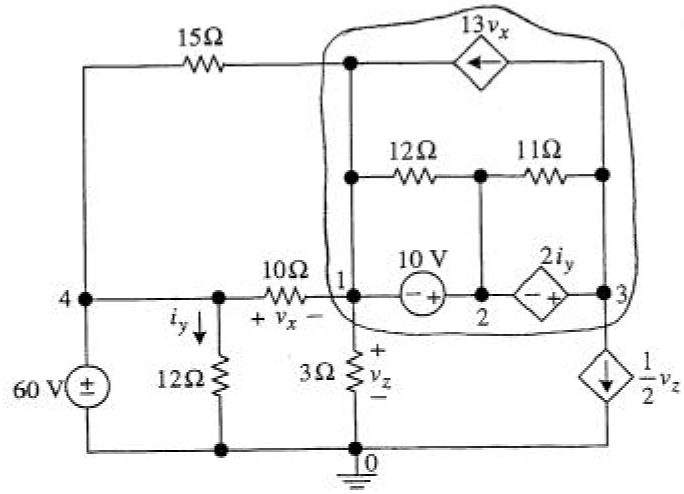
$C_{p2} = 14+6 = 20F$

$C_{s2} = \dfrac{1}{\frac{1}{30}+\frac{1}{20}} = 12F$

2. Write a matrix node-voltage equation for the circuit drawing shown on the right. Unknowns v_x, i_y, and v_z **must not** appear in the final matrix equation.

\# KCL eqs.

$= 5-1-3 = 1$



$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{6} \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & \frac{1}{6} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \\ 0 \\ 60 \end{bmatrix}$$

KCL nodes 1, 2, 3

$$\frac{1}{3}v_1 + \frac{1}{10}(v_1 - v_4) + \frac{1}{15}(v_1 - v_4) + \frac{1}{2}v_1 = 0$$

$$\left(\frac{1}{3} + \frac{1}{10} + \frac{1}{15} + \frac{1}{2}\right)v_1 - \left(\frac{1}{10} + \frac{1}{15}\right)v_4 = 0$$

$$v_1 - \frac{1}{6}v_4 = 0$$

KVL nodes 1 and 2

$$v_1 - v_2 = -10$$

KVL nodes 2 and 3

$$v_2 - v_3 = -2\left(\frac{1}{12}v_4\right)$$

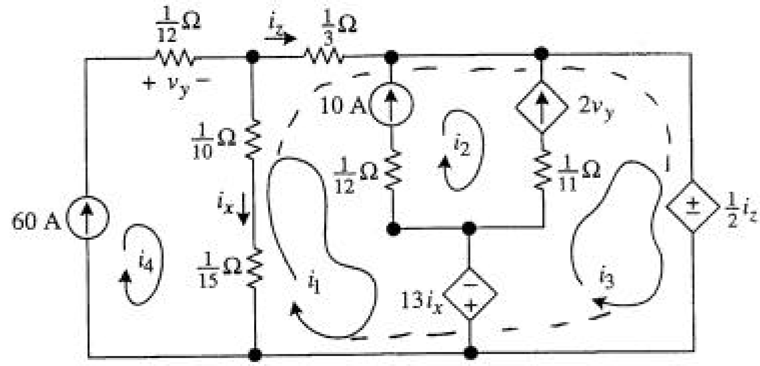
$$v_2 - v_3 + \frac{1}{6}v_4 = 0$$

KVL node 4

$$v_4 = 60$$

3. Write a matrix mesh-current equation for the circuit at the right. Unknowns i_x, v_y, and i_z **must not** appear in the final matrix equation.

\# KVL eqs. $= 4 - 3 = 1$



$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{6} \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & \frac{1}{6} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \\ 0 \\ 60 \end{bmatrix}$$

KVL meshes 1, 2 and 3

$$\frac{1}{15}(i_1 - i_4) + \frac{1}{10}(i_1 - i_4) + \frac{1}{3} i_1 + \frac{1}{2} i_1 = 0$$

$$\left(\frac{1}{15} + \frac{1}{10} + \frac{1}{3} + \frac{1}{2}\right) i_1 - \left(\frac{1}{15} + \frac{1}{10}\right) i_4 = 0$$

$$1 i_1 - \frac{1}{6} i_4 = 0$$

KCL meshes 1 and 2.

$$i_1 - i_2 = -10$$

KCL meshes 2 and 3

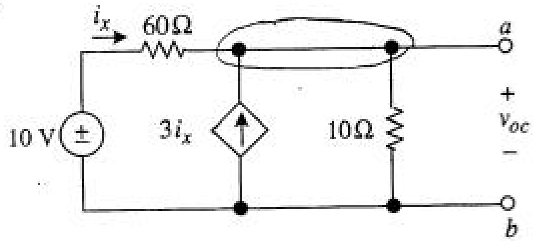
$$i_2 - i_3 = -2\left(\frac{1}{12} i_4\right)$$

$$i_2 - i_3 + \frac{1}{6} i_4 = 0$$

KCL mesh 4

$$i_4 = 60$$

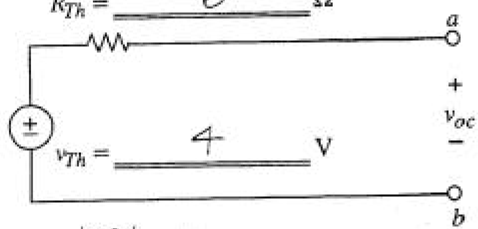
4.



Determine the Thevenin equivalent circuit with respect to terminals a and b.

R_{Th} = 6 Ω

v_{Th} = 4 V

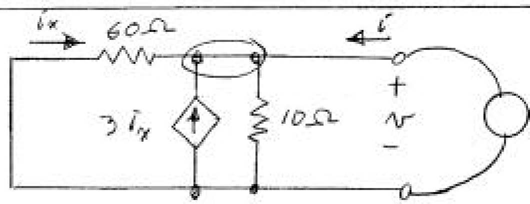


KCL

$$\frac{1}{60}(v_{oc}-10) - 3\left[\frac{1}{60}(10-v_{oc})\right] + \frac{1}{10}v_{oc} = 0$$

$$\left(\frac{1}{60}+\frac{3}{60}+\frac{6}{60}\right)v_{oc} = \frac{10}{60}+\frac{30}{60} = \frac{40}{60}$$

$$\boxed{v_{oc} = 4V}$$



KCL

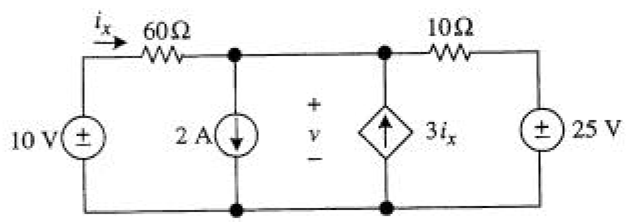
$$\frac{1}{60}v - 3\left[-\frac{1}{60}v\right] + \frac{1}{10}v - i = 0$$

$$\frac{1+3+6}{60}v = i$$

$$v = 6i = R_{Th}\, i$$

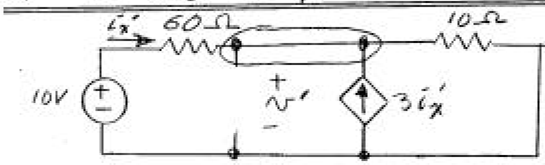
$$\boxed{R_{Th} = 6\Omega}$$

5.



Use **superposition** to calculate voltage *v*.

$v = 4 - 12 + 15 = 7$ V

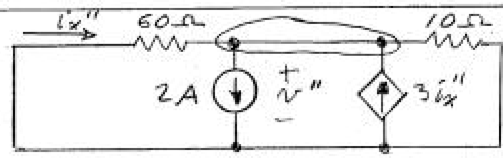


KCL

$$\frac{1}{60}(v'-10) - 3\left[\frac{1}{60}(10-v')\right] + \frac{1}{10}v' = 0$$

$$\left(\frac{1+3+6}{60}\right)v' = \frac{10+30}{60} = \frac{40}{60}$$

$$\boxed{v' = 4\,V}$$

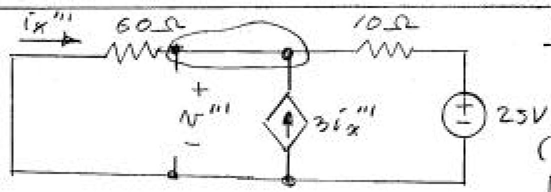


KCL

$$\frac{1}{60}v'' + 2 - 3\left[-\frac{1}{60}(v'')\right] + \frac{1}{10}v'' = 0$$

$$\left(\frac{1+3+6}{60}\right)v'' = -2$$

$$\boxed{v'' = -12\,V}$$



KCL

$$\frac{1}{60}v''' - 3\left[-\frac{1}{60}(v''')\right] + \frac{1}{10}(v'''-25) = 0$$

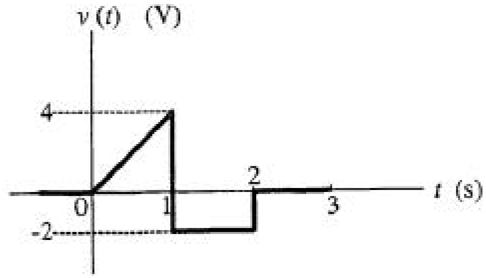
$$\left(\frac{1+3+6}{60}\right)v''' = \frac{25}{10} = \frac{5}{2}$$

$$\boxed{v''' = 15\,V}$$

$$v = v' + v'' + v''' = 4 - 12 + 15$$

$$\boxed{v = 7\,V}$$

6.



a) Write voltage $v(t)$ in terms of the unit step and/or the unit ramp functions.

$v(t) = \underline{4r(t) - 4r(t-1) - 6u(t-1) + 2u(t-2)}$ V

$= 4t\,u(t) - 4(t-1)u(t-1) - 6u(t-1) + 2u(t-2)$

$= 4t\,u(t) - (4t+2)\,u(t-1) + 2\,u(t-2)$

The voltage shown in the graph represents one period of a periodic voltage with period $T = 3$ s

b) Determine the average voltage (the dc component of $v(t)$).

$V_{av} = V_{dc} = \underline{\quad 0 \quad}$ V

$V_{av} = V_{dc} = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{3}\left[\int_0^1 4t\,dt + \int_1^2 (-2)\,dt\right]$

$= \frac{1}{3}\left[\frac{4t^2}{2}\Big|_0^1 - 2t\Big|_1^2\right] = \frac{1}{3}[2 - 2] = 0\ V$

c) Determine the effective (*rms*) value of $v(t)$.

$V_{eff} = V_{rms} = \underline{\frac{2}{3}\sqrt{7} = 1.764}$ V

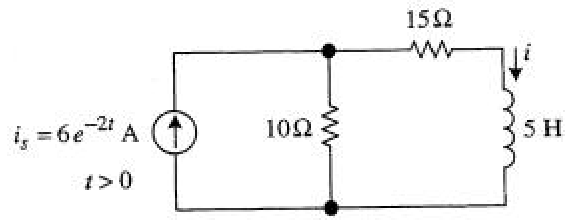
$V_{eff} = V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = \sqrt{\frac{1}{3}\left[\int_0^1 16t^2\,dt + \int_1^2 4\,dt\right]}$

$= \sqrt{\frac{1}{3}\left[\frac{16}{3}t^3\Big|_0^1 + 4t\Big|_1^2\right]} = \sqrt{\frac{1}{3}\left[\frac{16}{3} + 4\right]}$

$= \sqrt{\frac{28}{9}} = \frac{1}{3}\sqrt{4(7)} = \frac{2}{3}\sqrt{7}$

$= 1.7638\ V$

7. Determine $i(t)$ for $t > 0$.



$i(0^+) = 10\,\text{A}$

$i(t) = 4e^{-2t} + 6e^{-5t}$ A $t > 0$

KVL $\quad 5\frac{d}{dt}i + 10(i - i_s) + 15i = 0$

$5\frac{d}{dt}i + 25i = 10i_s$

$\frac{d}{dt}i + 5i = 2(6e^{-2t})$

$s + 5 = 0 \Rightarrow s = -5 \Rightarrow \boxed{i_n = Ae^{-5t}}$

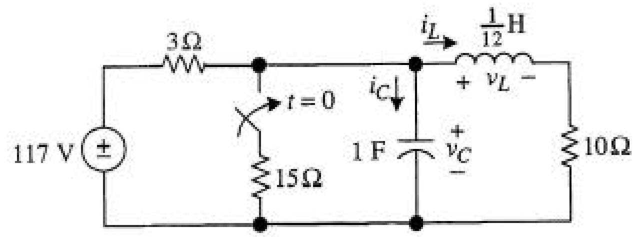
$-2i_p + 5i_p = 2(6e^{-2t}) \Rightarrow \boxed{i_p = 4e^{-2t}\ A}$

$i = i_p + i_n \Rightarrow \boxed{i = 4e^{-2t} + Ae^{-5t}}$

$i(0^+) = 10 = 4e^{-2(0)} + Ae^{-5(0)} \Rightarrow \boxed{A = 6}$

$\boxed{i = 4e^{-2t} + 6e^{-5t}\ A \quad t > 0}$

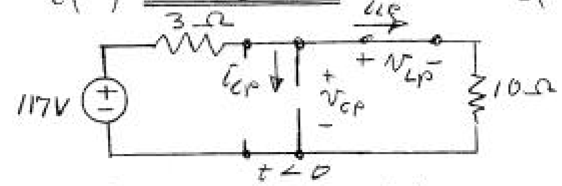
8.



Determine:

a) $v_C(0^-) =$ 90 V $\qquad v_L(0^-) =$ 0 V

$i_C(0^-) =$ 0 A $\qquad i_L(0^-) =$ 9 A



KVL

$-117 + 3i_{LP} + 10i_{LP} = 0$

$i_{LP} = \frac{117}{13} = 9A$

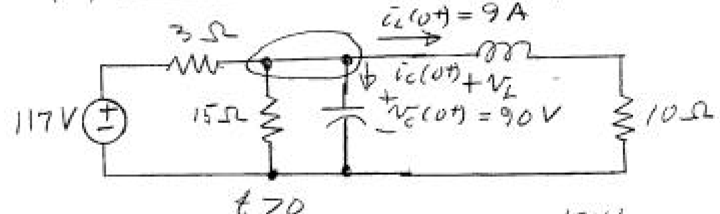
$= i_L(0^-)$

$v_{CP} = 10\, i_{LP} = 10(9)$

$= 90V = v_C(0^-)$

b) $v_C(0^+) =$ 90 V $\qquad v_L(0^+) =$ 0 V

$i_C(0^+) =$ −6 A $\qquad i_L(0^+) =$ 9 A



$v_c(0^+) = v_c(0^-) = 90V$

$i_L(0^+) = i_L(0^-) = 9A$

KVL

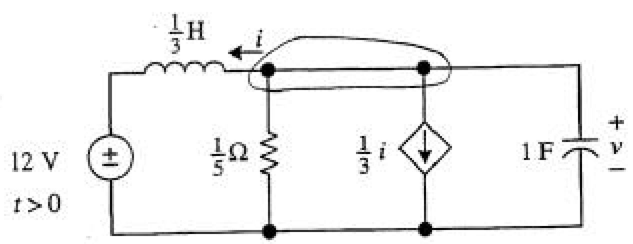
$v_L = 90 - 10(9) = 0$

KCL

$\frac{1}{3}(90-117) + \frac{1}{15}(90) + i_c(0^+) + 9 = 0$

$i_c(0^+) = -6A$

9.



$v(0^+) = 3$ V

$i(0^+) = 9$ A

Calculate $v(t)$ for $t > 0$.

$v(t) = \underline{12 - 21e^{-t} + 12e^{-4t}}$ V $\quad t > 0$

<u>KCL</u> $\quad 3\int_{-\infty}^{t}(v-12)\,d\lambda + 5v + \frac{1}{3}\left[3\int_{-\infty}^{t}(v-12)\,d\lambda + 1\frac{d}{dt}v\right] = 0$

$$\frac{d^2}{dt^2}v + 5\frac{d}{dt}v + 4v = 48$$

$$0^2 v_p + 5(0)v_p + 4v_p = 48 \Rightarrow \boxed{v_p = 12V}$$

$$s^2 + 5s + 4 = 0 \Rightarrow s = -1, -4 \Rightarrow \boxed{v_h = A_1e^{-t} + A_2e^{-4t}}$$

$$v = v_p + v_h \Rightarrow \boxed{v = 12 + A_1e^{-t} + A_2e^{-4t}}$$

$$v(0^+) = 3 = 12 + A_1 + A_2 \Rightarrow \boxed{A_1 + A_2 = -9}$$

<u>Evaluate the original KCL equation at $t = 0^+$</u>

$$9 + 5(3) + \frac{1}{3}(9) + \frac{d}{dt}\left[12 + A_1e^{-t} + A_2e^{-4t}\right]\Big|_{t=0^+} = 0$$

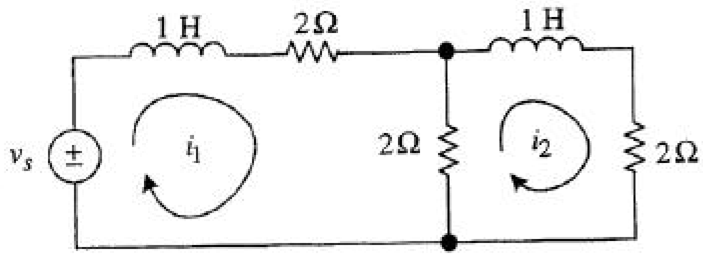
$$27 - A_1 - 4A_2 = 0 \Rightarrow \boxed{A_1 + 4A_2 = 27}$$

$$\begin{bmatrix}1 & 1\\ 1 & 4\end{bmatrix}\begin{bmatrix}A_1\\ A_2\end{bmatrix} = \begin{bmatrix}-9\\ 27\end{bmatrix}$$

$$A_1 = \frac{\begin{vmatrix}-9 & 1\\ 27 & 4\end{vmatrix}}{\begin{vmatrix}1 & 1\\ 1 & 4\end{vmatrix}} = \frac{-36-27}{3} = -21$$

$$A_2 = \frac{\begin{vmatrix}1 & -9\\ 1 & 27\end{vmatrix}}{3} = \frac{27+9}{3} = 12$$

10.



Determine the natural-response component of mesh current i_1.

$i_{1n}(t) =$ $A_1 e^{-2t} + A_2 e^{-6t}$ A

KVL mesh 1

$$-v_s + \frac{d}{dt} i_1 + 2 i_1 + 2(i_1 - i_2) = 0$$

$$(D+4) i_1 - 2 i_2 = v_s$$

KVL mesh 2

$$\frac{d}{dt} i_2 + 2 i_2 + 2(i_2 - i_1) = 0$$

$$-2 i_1 + (D+4) i_2 = 0$$

$$\begin{bmatrix} D+4 & -2 \\ -2 & D+4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} D+4 & -2 \\ -2 & D+4 \end{vmatrix} i_1 = \begin{vmatrix} v_s & -2 \\ 0 & D+4 \end{vmatrix}$$

$$[(D+4)(D+4) - (-2)(-2)] i_1 = (D+4) v_s$$

$$(D^2 + 8D + 12) i_1 = (D+4) v_s$$

$$s^2 + 8s + 12 = 0 \implies s = -2, -6$$

$$i_{1n} = A_1 e^{-2t} + A_2 e^{-6t}$$