KEY

NAME________________________

Instructor/College________________________

Section________________________

Score____________

EE 153 FINAL EXAM

WINTER SEMESTER 2000

CLOSED BOOK

2 HOUR TIME LIMIT

CALCULATORS ARE ALLOWED

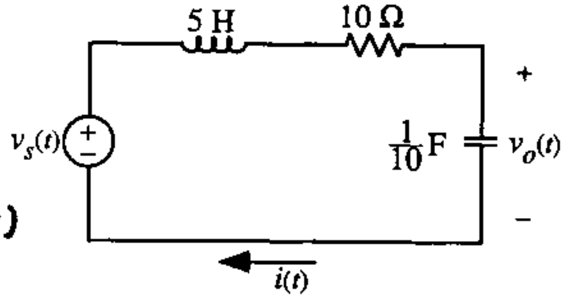
There are 12 problems; please look over your exam to make sure you have 12 different problems. **Do any ten (10) problems!** Draw a large X through the two problems that you do not want to be graded. If you do not indicate which problems you want to leave out, the first 10 problems will be graded.

Do all work for each problem only on the page(s) supplied for that problem. **DO NOT,** for instance, continue Problem 3 on the back of Problem 2. Extra blank paper will be supplied if needed. If extra paper is used, show the additional work for each problem on a separate sheet and staple the extra sheet(s) to the appropriate problem(s).

ALL PHASORS WILL BE IN BOLD FACE TYPE

(1) Consider the following circuit driven by a voltage source, $v_s(t) = 100\cos(2t + 30°)$ V. Redraw the circuit in frequency domain. Calculate voltage $v_o(t)$ and current $i(t)$. Show the voltage phasors $\mathbf{V}_s$, $\mathbf{V}_o$, and $\mathbf{I}$ on phasor diagram. Write differential equation relating v_o and v_s. Find transfer function $H(j\omega) = \mathbf{V}_o / \mathbf{V}_s$.

USING $\omega = 2$



$$\hat{I} = \frac{100\angle 30°}{j10 + 10 - j5} = \frac{100\angle 30°}{11.18\angle 26.56°} = 8.94\angle 3.44° \text{ A}$$

$$\hat{V}_o = \left(\frac{-j5}{10+j5}\right)(100\angle 30°) = \left(\frac{5\angle -90°}{11.18\angle 26.56}\right)(100\angle 30°)$$

$$= 42.37\angle -86.56° \text{ V}$$

$$H(j\omega) = \frac{-j10/\omega}{j\omega 5 + 10 - j10/\omega} = \frac{10}{5(j\omega)^2 + 10(j\omega) + 10} = \frac{2}{(j\omega)^2 + 2(j\omega) + 2} = \frac{\hat{V}_o}{\hat{V}_s}$$

$$\left[(j\omega)^2 + 2(j\omega) + 2\right]\hat{V}_o = 2\hat{V}_s$$

$$(D^2 + 2D + 2)v_o = 2v_s$$

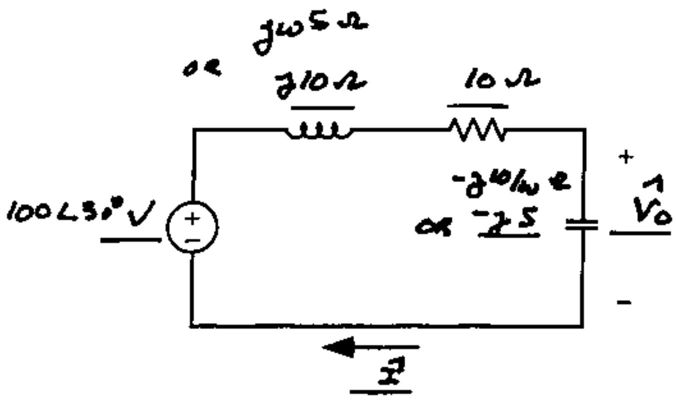
OR $H(j2) = \frac{2}{-4 + j4 + 2} =$

$$\frac{2}{-2+j4} = \frac{1}{-1+j2} = .449\angle -116.$$

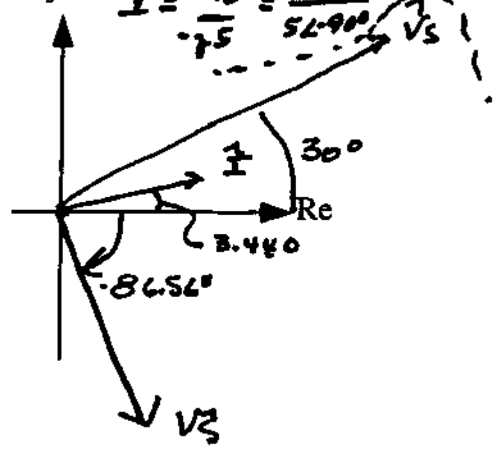
$$\hat{V}_o = H(j2)\hat{V}_s = (.449\angle -116.56)(100\angle$$

$$= 44.9\angle -86.56° \text{ V}$$

Answer:



$$\hat{I} = \frac{\hat{V}_o}{-j5} = \frac{44.9\angle -86.56}{5\angle -90°} = 8.94\angle$$



$v_o(t) =$ $42.97\cos(2t - 86.56°)$ V

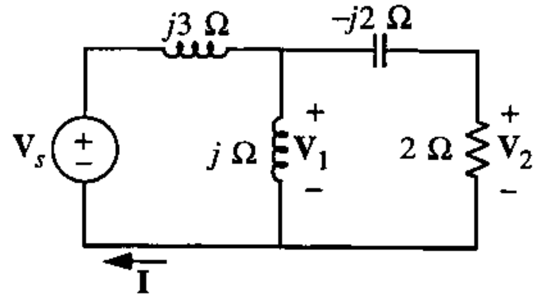
$i(t) =$ $8.94\cos(2t + 3.44°)$ A

$(D^2 + 2D + 2)v_o = (2)v_s$

$H(j\omega) = \dfrac{2}{(j\omega)^2 + 2j\omega + 2}$

Problem Score

(2) For the following circuit, find $\mathbf{H}_{1s}(j10) = \mathbf{V}_1 / \mathbf{V}_s$, $\mathbf{H}_{21}(j10) = \mathbf{V}_2 / \mathbf{V}_1$, and $\mathbf{H}_{2s}(j10) = \mathbf{V}_2 / \mathbf{V}_s$. Given that $v_s(t) = 100\cos(10t + 45°)$ V, find $v_2(t)$. The values of the impedances have been calculated using $\omega = 10$ rad / s in the diagram below.



$$j1 \| (2-j2) = \frac{(1\angle 90°)(2\sqrt{2}\angle -45°)}{2-j}$$

$$= \frac{2\sqrt{2}\angle 45°}{\sqrt{5}\angle -26.56°} = 1.26\angle 71.56°$$

$$= .4 + j1.2$$

$$H_{1s}(j10) = \frac{\vec{V}_1}{\vec{V}_s} = \frac{1.26\angle 71.56°}{.4 + j1.2 + j3} = \frac{1.26\angle 71.56°}{.4 + j4.2} = \frac{1.26\angle 71.56°}{4.28\angle 84.56°} = .3\angle -13°$$

$$H_{21}(j10) = \frac{\vec{V}_2}{\vec{V}_1} = \frac{2}{2-j2} = \frac{1}{1-j} = \frac{1\angle 0°}{\sqrt{2}\angle -45°} = .71\angle 45°$$

$$H_{2s}(j10) = H_{21}(j10)\, H_{1s}(j10) = (.3\angle -13°)(.71\angle 45°) = .213\angle 32°$$

$$\vec{V}_2 = H_{2s}(j10)\, \vec{V}_s = (.213\angle 32°)(100\angle 45°) = 21.3\angle 77°$$

Answer:

$\mathbf{H}_{1s}(j10) =$ $.3\angle -13°$

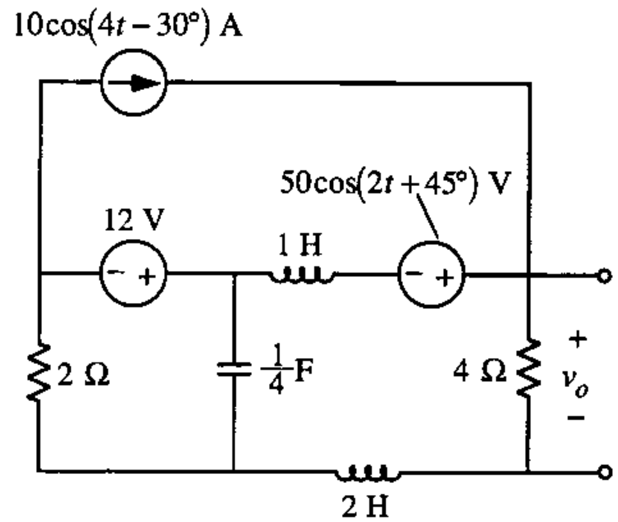
$\mathbf{H}_{21}(j10) =$ $.71\angle 45°$

$\mathbf{H}_{2s}(j10) =$ $.213\angle 32°$

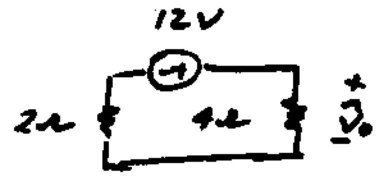
$v_2(t) =$ $21.3\cos(10t + 77°)$ V

Problem Score

(3) Find $v_o(t)$ for the following circuit.



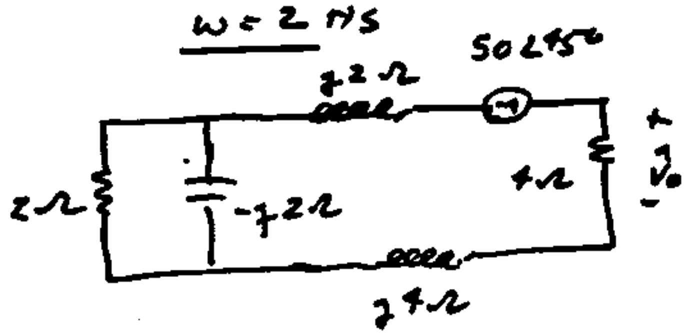
$\underline{\omega = 0 \text{ r/s}}$



$\hat{V}_o = \left(\frac{4}{6}\right)(12)$

$= 8$

$\underline{\omega = 2 \text{ r/s}}$



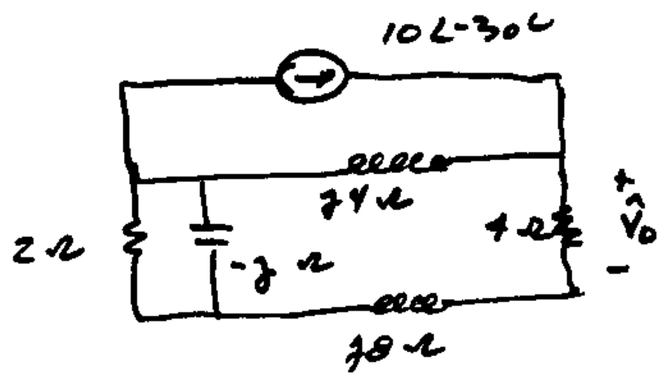
$2 \| -j2 = \dfrac{(2\angle 0°)(2\angle -90°)}{2\sqrt{2}\angle -45°} = \sqrt{2}\angle -45°$

$= 1 - j$

$\hat{V}_o = \left(\dfrac{4}{j2 + 1 - j + j4 + 4}\right)(50\angle 45°) = \dfrac{200\angle 45°}{5 + j5} = \dfrac{200\angle 45°}{5\sqrt{2}\angle 45°} = 20\sqrt{2}\angle 0°$

$= 28.28\angle 0°$

$\underline{\omega = 4 \text{ r/s}}$



$2 \| -j = \dfrac{(2\angle 0°)(1\angle -90°)}{2 - j} = \dfrac{2\angle -90°}{2.23\angle -26.6°}$

$= .896\angle -63.4° = .40 - j.80$

$\hat{V}_o = 4\left(\dfrac{j4}{j4 + 4 + j8 + .4 - j.8}\right)(10\angle -30°)$

$= \dfrac{4(4\angle 90°)(10\angle -30°)}{4.4 + j11.2} = \dfrac{160\angle 60°}{12.03\angle 68.5} = 13.3\angle -8.5°$

Answer:

$v_o(t) = \underline{8 + 28.28\cos(2t)} + 13.3\cos(4t - 8.5°)$ V

Problem Score

(4) A 240 V_{rms} single phase 60 Hz system is connected to a 20 hp motor that is 95% efficient and has a power factor of 0.8 lagging. Draw the power triangle. Find line current, I_{L1}. A second load of either an inductor or a capacitor is added in parallel to the motor to make the power factor equal to 1. Find the size of the added capacitor or inductor and the line current, I_L of the combined load. (745.7 watt = 1 hp)

$$P = \frac{(20)(745.7)}{.95} = 15700\,w$$

$$I_{L_1} = \frac{19625}{240} = 81.8\ A_{rms}$$

$$Q = -11775\ VAC$$

$$I_{L_2} = \frac{Q}{V} = \frac{11775}{240} = 49.1A$$

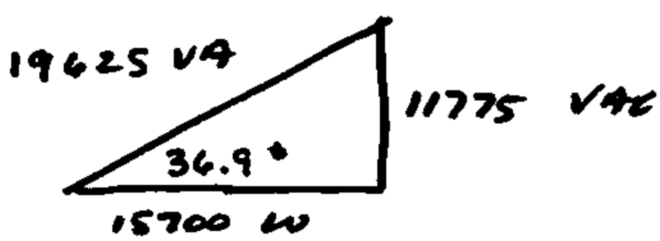
$$|Z| = \frac{240}{49.1} = 4.89\,\Omega = \frac{1}{377C}$$

$$C = \frac{1}{(377)(4.89)} = .000542\,F$$

$$I_L = \frac{15700}{240} = 65.4\ A_{rms}$$

Answer:

Power Triangle



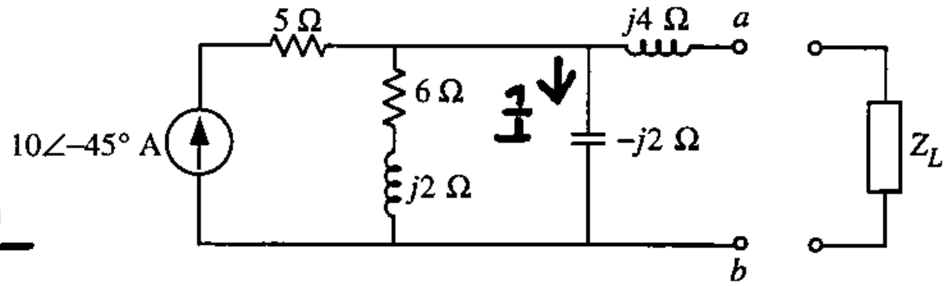
I_{L1} = 81.8 Arms

$\boxed{C}$ or L = 542 μF

I_L = 65.4 Arms

Problem Score

(5) For the following circuit, find the Thévenin equivalent current. What load Z_L should be added across terminals *a-b* so that maximum power is delivered to the load? What is the maximum transfer power, $p_{L(\max)}$ to the above load?



$\hat{I} = \left(\frac{6+j2}{6}\right)(10\angle -45°)$

$= \frac{(6.32\angle 18.4°)(10\angle -45°)}{6}$

$= 10.53\angle -26.6°$

$\hat{V}_{oc} = \hat{V}_{TH} = (-j2)\hat{I} = (2\angle -90°)(10.53\angle -26.6°) = 21.06\angle -116.6°$ V

$Z_{TH} = (6+j2)\,||\,(-j2) + j4 = \frac{(6.32\angle 18.4°)(2\angle -90°)}{6} + j4$

$= 2.1\angle -71.6° + j4 = .66 - j2 + j4 = .66 + j2 = 2.1\angle 71.70$

$Z_L = Z_{TH}^* = .66 - j2 = 2.1\angle -71.7°$

$|I_L| = \frac{21.06}{2(.66)} = 15.95 A$

$P_L = \frac{1}{2} I_L^2 R = \frac{1}{2}(15.95)^2(.66) = 83.9$ W

Answer:

$Z_{TH} =$ $2.1\angle 71.7° = .66 + j2$ Ω

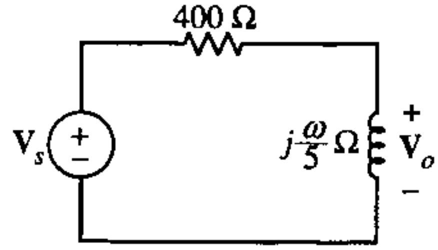
$V_{TH} =$ $21.06\angle -116.6°$ V

$Z_L =$ $2.1\angle -71.7° = .66 - j2$ Ω

$p_{L(\max)} =$ 83.9 W

Problem Score

(6) Find the transfer function $\mathbf{H}(j\omega)=\dfrac{\mathbf{V}_o}{\mathbf{V}_s}$. Graph $|\mathbf{H}|$ vs ω. Find half power cutoff frequency, ω_c. Determine if it is a high pass, low pass, band pass or band reject filter. Find $v_o(t)$ if $v_s(t)=100\cos(1000t+30°)\,\text{V}$.



$$H(j\omega)=\frac{j\omega/5}{400+j\omega/5}=\frac{j\omega}{2000+j\omega}$$

$$=\frac{\omega\angle 90°}{\sqrt{(2000)^2+\omega^2}\;\angle\tan^{-1}\omega/2000}=$$

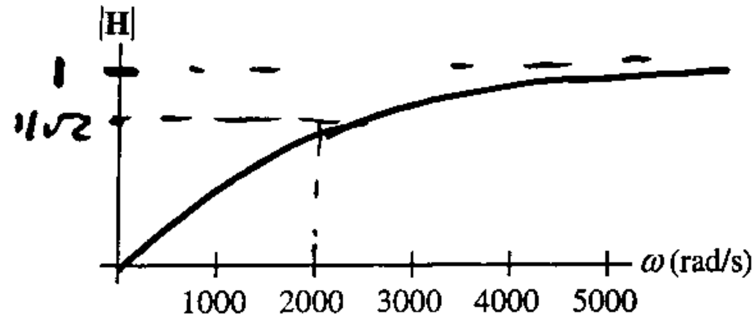
$$=\frac{\omega}{\sqrt{(2000)^2+\omega^2}}\angle 90°-\tan^{-1}\omega/2000$$

$$H(j1000)=\frac{1000}{\sqrt{4\times10^6+10^6}}\angle 90°-\tan^{-1}1/2=\frac{1}{\sqrt5}\angle 63.4°$$

$$\hat{V}_o=H(j1000)\,\vec{V}_s=\left(\frac{1}{\sqrt5}\angle 63.4°\right)(100\angle 30°)=44.7\angle 93.4°$$

Answer:

$\mathbf{H}(j\omega)=$ $\dfrac{j\omega}{2000+j\omega}=\dfrac{\omega}{\sqrt{(2000)^2+\omega^2}}\angle 90°-\tan^{-1}\omega/2000$



$\omega_c=$ 2000 r/s

<u>high pass</u> low pass band pass band reject

$v_o(t)=$ 44.7 cos(1000t + 93.4°) ✓

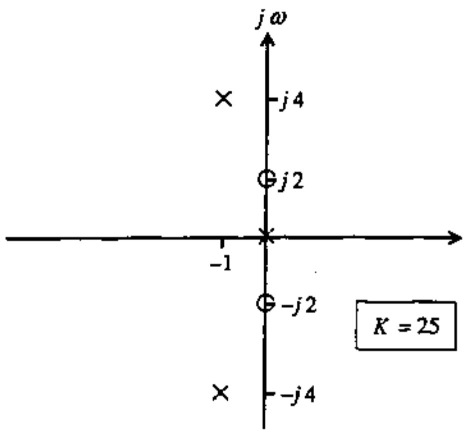
Problem Score

(7) The pole-zero plot shown below refers to an impedance, **Z**(s). Find **Z**(s), and the particular response, $v_p(t)$ for an input $i(t) = 6\cos(t + 45^\circ)\,A$.

Answer:

$$\mathbf{Z}(s) = 25\left[\frac{s^2+4}{s(s^2+2s+17)}\right]$$

$$v_p(t) = 27.9\cos(t+127.87^\circ)\,V$$



POLES

$s_{p_1} = 0$

$s_{p_2} = -1+j4$

$s_{p_3} = -1-j4$

ZEROS

$s_{z_1} = j2$

$s_{z_2} = -j2$

Simple Poles & Simple Zeros!

$$\boxed{\vec{Z}(s) = K\frac{(s-s_{z_1})(s-s_{z_2})}{(s-s_{p_1})(s-s_{p_2})(s-s_{p_3})}} \Rightarrow \vec{Z}(s) = 25\left[\frac{(s-j2)(s+j2)}{s(s+1-j4)(s+1+j4)}\right]$$

$$\Rightarrow \boxed{\vec{Z}(s) = 25\left[\frac{s^2+4}{s(s^2+2s+17)}\right]}$$

$$i(t) = 6\cos(t+45^\circ)A \Rightarrow s = j1, \quad \vec{I} = 6\angle 45^\circ A$$

$$\vec{Z}(j1) = 25\left[\frac{-1+4}{(j1)(-1+j2+17)}\right] = \frac{(25)(3)}{(j)2(j+8)} = \frac{15}{26}\sqrt{65}\,\angle\tan^{-1}(8)$$

$$\Rightarrow \vec{Z}(j1) = 4.65\angle 82.87^\circ$$

$$\boxed{\vec{V} = \vec{Z}(j1)\,\vec{I}}$$

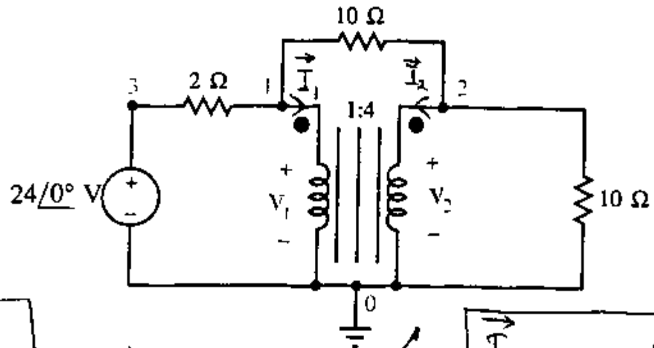
$$\Rightarrow \vec{V} = (4.65\angle 82.87^\circ)(6\angle 45^\circ)\,V$$

$$\Rightarrow \vec{V} = 27.9\angle 127.87^\circ\,V$$

$$\therefore \boxed{v_p(t) = 27.9\cos(t+127.87^\circ)\,V}$$

(8) Determine the node voltages, V_1 and V_2 in the following network.

$$\boxed{\frac{n_1}{n_2} = \frac{1}{4}}$$



Answer:

$V_1 =$ $4\angle 0^\circ$ V

$V_2 =$ $16\angle 0^\circ$ V

$$\boxed{\frac{\vec{V}_2}{\vec{V}_1} = \frac{n_2}{n_1}} \Rightarrow \boxed{\vec{V}_2 = 4\vec{V}_1}$$

$$\boxed{\frac{\vec{I}_2}{\vec{I}_1} = -\frac{n_1}{n_2}} \Rightarrow \boxed{\vec{I}_2 = -\frac{1}{4}\vec{I}_1}$$

KCL NODE 1 $\Rightarrow \dfrac{\vec{V}_1 - 24}{2} + \dfrac{\vec{V}_1 - \vec{V}_2}{10} + \vec{I}_1 = 0$

$\Rightarrow 5(\vec{V}_1 - 24) + (\vec{V}_1 - \vec{V}_2) + 10\vec{I}_1 = 0$

$\Rightarrow \boxed{2\vec{V}_1 + 10\vec{I}_1 = 120} \rightarrow$ 1

KCL NODE 2 $\Rightarrow \dfrac{\vec{V}_2}{10} + \dfrac{\vec{V}_2 - \vec{V}_1}{10} + \vec{I}_2 = 0$

$\Rightarrow 2\vec{V}_2 - \vec{V}_1 + 10\vec{I}_2 = 0$

$\Rightarrow 7\vec{V}_1 - \frac{10}{4}\vec{I}_1 = 0 \Rightarrow \boxed{28\vec{V}_1 = 10\vec{I}_1} \rightarrow$ 2

Using eq. 2 in 1

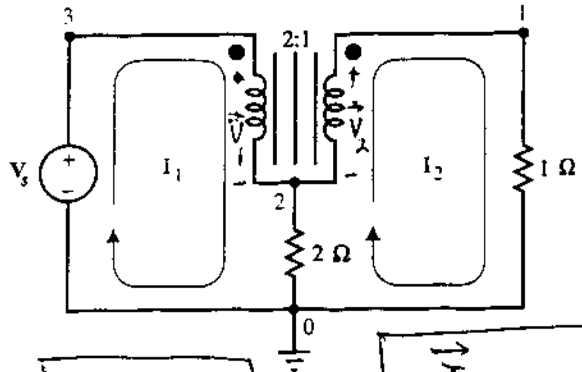
$2\vec{V}_1 + 28\vec{V}_1 = 120$

$\therefore\ \vec{V}_1 = 4\angle 0^\circ$ V

$\vec{V}_2 = 4\vec{V}_1 \Rightarrow \vec{V}_2 = 16\angle 0^\circ$ V

(9) Determine the mesh currents, I_1 and I_2 in the following circuit if $V_s = 120\angle 0^\circ$ V.

$$\boxed{\frac{n_1}{n_2} = \frac{2}{1}}$$



Answer:

$I_1 = 20\angle 0^\circ$ A

$I_2 = 40\angle 0^\circ$ A

$$\boxed{\frac{\vec{V}_2}{\vec{V}_1} = \frac{n_2}{n_1}} \Rightarrow \boxed{\vec{V}_2 = \frac{1}{2}\vec{V}_1}$$

$$\boxed{\frac{\vec{I}_2}{\vec{I}_1} = +\frac{n_1}{n_2}} \Rightarrow \boxed{\vec{I}_2 = 2\vec{I}_1}$$

Mesh ① KVL $\Rightarrow -\vec{V}_s + \vec{V}_1 + 2(\vec{I}_1 - \vec{I}_2) = 0$

$$\Rightarrow \boxed{\vec{V}_1 - 2\vec{I}_1 = 120} \longrightarrow ①$$

Mesh ② KVL $\Rightarrow 2(\vec{I}_2 - \vec{I}_1) - \vec{V}_2 + \vec{I}_2 = 0$

$$\Rightarrow 2\vec{I}_1 - \frac{1}{2}\vec{V}_1 + 2\vec{I}_1 = 0$$

$$\Rightarrow \boxed{\vec{V}_1 = 8\vec{I}_1} \longrightarrow ②$$

Using eq. ② in ①

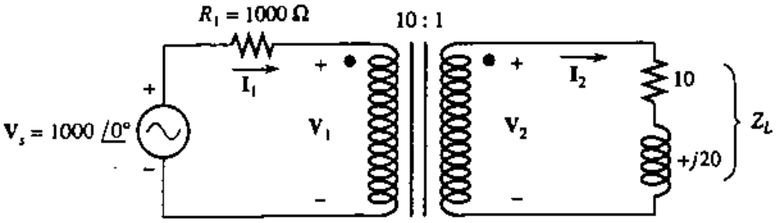
$$8\vec{I}_1 - 2\vec{I}_1 = 120$$

$$\therefore \vec{I}_1 = 20\angle 0^\circ \text{ A}$$

$$\vec{I}_2 = 2\vec{I}_1 \Rightarrow \vec{I}_2 = 40\angle 0^\circ \text{ A}$$

(10) For an ideal transformer shown below find the power, P_L delivered to the load.

$$\boxed{\frac{n_1}{n_2} = \frac{10}{1}}$$



Answer:

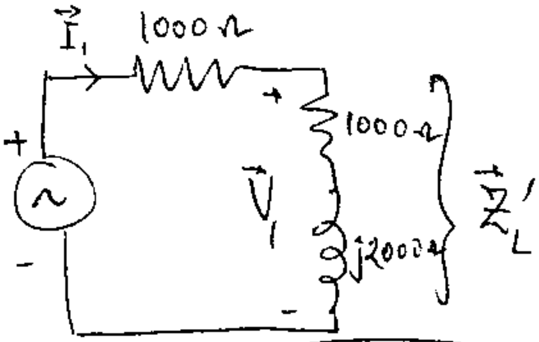
$P_L =$ 62.51 W

Reflect the load impedance, $\vec{Z}_L$ to the primary side of the transformer,

$$\boxed{\vec{Z}_L' = \left(\frac{n_1}{n_2}\right)^2 \vec{Z}_L}$$

$$\Rightarrow \vec{Z}_L' = \left(\frac{10}{1}\right)^2 (10 + j20)$$

$$\Rightarrow \vec{Z}_L' = 1000 + j2000$$



$\vec{V}_s = 1000\,\angle 0°\ \text{V}$

Total Impedance, $\boxed{\vec{Z}_s = R_1 + \vec{Z}_L'}$

$$\Rightarrow \vec{Z}_s = 1000 + 1000 + j2000$$

$$\Rightarrow \vec{Z}_s = 2000 + j2000 = 2828\,\angle 45°\,\Omega$$

$$\boxed{\vec{I}_1 = \frac{\vec{V}_s}{\vec{Z}_s}} \Rightarrow \vec{I}_1 = \frac{1000\,\angle 0°}{2828\,\angle 45°} = 0.3536\,\angle -45°\ \text{A}$$

$$\boxed{\vec{I}_2 = +\frac{n_1}{n_2}\vec{I}_1} \Rightarrow \vec{I}_2 = \left(\frac{10}{1}\right)\left(0.3536\,\angle -45°\right) = 3.536\,\angle -45°$$

Power delivered to the load, $\boxed{P_L = \left[I_{2(rms)}\right]^2 R_L}$

$$\Rightarrow P_L = \left(\frac{3.536}{\sqrt{2}}\right)^2 (10)\ \text{W}$$

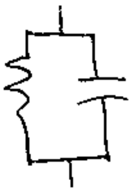
$$\Rightarrow P_L = 62.51\ \text{W}$$

(11) Each phase of a (wye)Y - connected load consists of a *50 Ω* resistance in parallel with a *100 μF* capacitance. Find the impedance of each phase of an equivalent (Delta) Δ - connected load $\boldsymbol{Z_\Delta}$. The frequency of operation is 60 Hz.

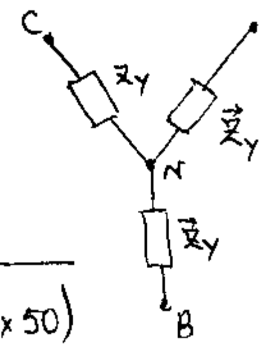
Answer:

$Z_\Delta = 70.29 \angle -62.05^\circ \, \Omega$

$$\left.\begin{aligned} R &= 50\,\Omega \\ C &= 100\,\mu F = 10^{-4} F \\ \omega &= 2\pi(60) \text{ rad/s} \end{aligned}\right\}$$



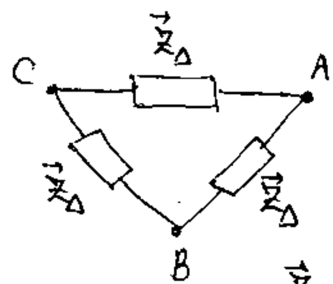
$$\vec{Z}_Y = R \,\|\, \frac{1}{j\omega C} = \frac{(R)\left(\frac{1}{j\omega C}\right)}{R + \frac{1}{j\omega C}}$$



$$\Rightarrow \boxed{\vec{Z}_Y = \frac{R}{1 + j\omega C R}} \Rightarrow \vec{Z}_Y = \frac{50}{1 + j(2\pi \times 60 \times 10^{-4} \times 50)}$$

$\vec{Z}_Y = \vec{Z}$

$$\Rightarrow \vec{Z}_Y = \frac{50}{1 + j\pi(0.6)}$$



$$\Rightarrow \vec{Z}_Y = 23.43 \angle -62.05^\circ \, \Omega$$

$\vec{Z}_\Delta = \vec{Z}_L$

$$\boxed{\vec{Z}_\Delta = 3\vec{Z}_Y} \Rightarrow \vec{Z}_\Delta = 70.29 \angle -62.05^\circ \, \Omega$$

(12) A balanced three-phase source serves three loads as follows:

Load 1: 24 kW at 0.6 lagging power factor

Load 2: 10 kW at unity power factor

Load 3: 12 kVA at 0.8 leading power factor

If the line voltage at the loads is 208 V rms at 60 Hz, determine the line current, I_L and the combined power factor of the loads, PF_{load}.

Answer:

$I_L = 139.28$ A (rms)

$PF_{load} = 0.869$, lagging

$P_1 = 24{,}000$ W

$PF_1 = 0.6$, lagging $\Rightarrow \cos\theta_1 = 0.6$

$\Rightarrow Q_1 = P_1 \tan\theta_1 = 32{,}000$ VAR

$\Rightarrow \boxed{\vec{S}_1 = 24{,}000 + j32{,}000 \text{ VA}}$

$P_2 = 10{,}000$ W

$PF_2 = 1.0 \Rightarrow \cos\theta_2 = 1.0$

$\Rightarrow Q_2 = P_2 \tan\theta_2 = 0$

$\Rightarrow \boxed{\vec{S}_2 = 10{,}000 + j0 \text{ VA}}$

$PF_3 = 0.8$, leading $\Rightarrow \theta_3 = -36.9°$

$\vec{S}_3 = 12{,}000 \angle{-36.9°}$ VA $\Rightarrow \boxed{\vec{S}_3 = 9600 - j7200 \text{ VA}}$

$\boxed{\vec{S}_{Load} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3}$

$\Rightarrow \boxed{\vec{S}_{Load} = 43{,}600 + j24{,}800 = 50{,}160 \angle 29.63° \text{ VA}}$

$I_L = \dfrac{|\vec{S}_{Load}|}{\sqrt{3}\, V_L} \Rightarrow I_L = \dfrac{50{,}160}{\sqrt{3}(208)}$ A (rms)

$\Rightarrow I_L = 139.28$ A (rms)

$PF_{load} = \cos 29.63°$

$\Rightarrow PF_{load} = 0.869$, lagging