Electrical Engineering Advancement Exam III

SPRING SEMESTER 2021

CLOSED BOOK, CLOSED NOTES
2 HOUR TIME LIMIT

CALCULATORS ARE ALLOWED
(calculators without communication capability only)

ELECTRONIC DEVICES WITH COMMUNICATION CAPABILITY
(electronic devices such as cell phone, pagers, and iPads)

MAY NOT BE USED DURING THE EXAMINATION
(If such devices ring or are visible, a 10% penalty will be given for the first occurrence
and exam failure for the second.)

There are 10 problems: please look over the exam to make sure that you have 10
different problems. **Do any eight (8) problems!** Draw a large X through the two
problems that you do not want to be graded. If you do not indicate which problems you
want to leave out, the first 8 problems will be graded.

Do all work for each problem only on the page supplied for that problem (you may
use both sides). **DO NOT**, for instance, continue Problem #3 on the back of Problem #2.
Extra blank paper will be supplied if needed. If extra paper is used, show the additional
work for each problem on a separate sheet and staple the extra sheet(s) to the appropriate
problems.
For parts a, b, and c, consider a GaAs semiconductor ($n_i = 2.1 \times 10^6$ cm$^{-3}$ at RT) doped with only one type of unknown dopant that results in the shown energy band diagram. Assume RT for the numerical analysis.

(a) [3 points] Is the semiconductor n-type or p-type? (Circle one)

- n-type
- p-type

Answer: n-type

(b) [4 points] Identify the types of dopants that could possibly create this semiconductor. (Circle all possible cases.)

- Ge
- Si
- P
- N
- In

A: Ge Si

(c) [10 points] If $E_C - E_F = 0.25$ eV, calculate the majority and minority concentration values. Go up to 7 decimal places for all answers.

**Answer:**

- $n_o = n_i \exp \left[\frac{(E_F - E_i)}{kT}\right] = (2.1 \times 10^6) \exp \left[\frac{0.25}{0.0259}\right] = 1.0853194 \times 10^{14}$ cm$^{-3}$
- $p_o = n_i \exp \left[\frac{-(E_F - E_i)}{kT}\right] = (2.1 \times 10^6) \exp \left[-\frac{0.25}{0.0259}\right] = 0.0406332$ cm$^{-3}$

(d) [2 points] For an **intrinsic** semiconductor, sketch the variation of $n_i$ vs. $1/T$ where $T$ is temperature.

(e) [6 points] For a doped semiconductor, sketch the variation of the majority carrier vs. $1/T$ where $T$ is temperature. Clearly label the intrinsic and extrinsic region.
Consider the energy band diagram of a biased Si PN junction shown. An external bias voltage \( V_A \) is applied per our convention. The RT intrinsic concentration of Si is \( 1.50 \times 10^{10} \text{ cm}^3 \).

\[ E_{FP} - E_{FN} = 0.40 \text{ eV} \]
\[ E_{FN} - E_{IN} = 0.20 \text{ eV} \]

Assuming room temperature.

(a) [3 points] Is the junction reverse biased or forward biased? (Circle one)

- Reverse Biased
- Forward Biased

Answer: Reverse Biased

(b) [16 points] Calculate the values of the effective doping on each side of the junction, of the contact potential \( V_0 \), and of the external bias voltage \( V_A \) if the observed value of depletion width is \( 9.0338 \mu \text{m} \). Assume that the effective doping concentration, in each region, is equal to the majority carrier concentration in that region.

\[ V_0 = (1/q)[(E_{FN} - E_i) - (E_{FP} - E_i)] = (1/q)[0.2 \text{ eV} - (-0.4 \text{ eV})] = 0.6 \text{ V} \]

\[ N_{\text{dn,eff}} = n_o = n_i \exp[(E_{FN} - E_i)/kT] = (1.5 \times 10^{10}) \exp[0.2/0.0259] = 3.3862132 \times 10^{13} \text{ cm}^3 \]

\[ N_{\text{dp,eff}} = p_o = n_i \exp[-(E_{FP} - E_i)/kT] = (1.5 \times 10^{10}) \exp[-0.4/0.0259] = 7.6443744 \times 10^{16} \text{ cm}^3 \]

\( W = \) (see equation sheet)

\[ V_A = -1.5 \text{ V} \]

(c) [3 points] Express the symbolic values of the energies marked “A”, “B” and “C” in the energy band diagram shown above in terms of the parameters \( V_0 \) and \( V_A \).

Answers: \( A = qV_A \) \quad \( B = q(V_0 - V_A) \) \quad \( C = q(V_0 - V_A) \)

\[ A = \text{ eV} \quad B = \text{ eV} \quad C = \text{ eV} \]

(d) [3 points] Consider this pn junction unbiased. Will the depletion region extend more into the p-side or the n-side, i.e. which is greater \( x_{no} \) or \( x_{po} \)? (Circle one)

Answer: \( x_{no} > x_{po} \)

\[ x_{no} > x_{po} \quad \quad x_{no} < x_{po} \]
Consider the diode circuit shown. Each diode has a turn-on voltage of 0.60 V and reverse saturation current of 0 A. The voltage $V_R = 4.0$ V and the resistor $R = 2.0 \, \text{k}\Omega$

(a) [6 Points] If $V_A = V_B = 6.0 \, \text{V}$, Calculate $V_o$, $I_a$, and $I_b$.

**Answers:**

- $V_A = V_B = 6 \, \text{V}$ and $V_R = 4 \, \text{V}$
- Diodes Reverse Biased
- $I_A = I_B = I_o = 0 \, \text{A}$
- KVL: $-4 + IR = V_o$ gives $V_o = 4 \, \text{V}$

(b) [8 Points] If $V_A = 0 \, \text{V}$ and $V_B = 6.0 \, \text{V}$, Calculate $V_o$, $I_a$, and $I_b$.

**Answers:**

- $V_A = 0 \, \text{V}$ and $V_B = 6 \, \text{V}$ and $V_R = 4 \, \text{V}$
- Diode A Forward Biased and Diode B Reverse Biased $I_b = 0$
- $-V_o + V_{dA} + V_A = 0$ and $V_o = 0.6 \, \text{V}$
- KVL: $-4 + IR = V_o$ gives $I_a = 1.7 \, \text{mA}$

(c) [8 Points] If $V_A = 0 \, \text{V}$ and $V_B = 0 \, \text{V}$, Calculate $V_o$, $I_a$, and $I_b$.

**Answers:**

- $V_A = V_B = 0 \, \text{V}$ and $V_R = 4 \, \text{V}$
- Diodes Forward Biased
- $-V_o + V_{dA} + V_{A,B} = 0$ and $V_o = 0.6 \, \text{V}$
- KVL: $-4 + IR = V_o$ gives $I = 1.7 \, \text{mA}$ and $I_{a,b} = 1/2 = 0.85 \, \text{mA}$

(d) [8 Points] What type of system does the above circuit represent? (Circle one)

- OR Gate
- Clamper
- XOR Gate
- None of those listed

**Answers:** None Listed (AND Gate)

Problem Score /25
The two circuits shown need to have equivalent operating points. The circuit values are

\[ V_{CC} = 15.0 \text{ V}, \quad V_{BB} = 5.00 \text{ V}, \]
\[ R_b = 50.0 \text{ k}\Omega, \quad R_e = 0 \text{ \Omega}, \quad \text{and} \quad R_C = 500 \text{ \Omega}. \]

Assume a gain (nominal) of \( \beta = 100 \) and a base-emitter turn-on voltage of \( V_{to} = 0.70 \text{ V} \).

(a) [10 points] Calculate the required values of \( R_1 \) and \( R_2 \) for the circuits to be equivalent.

**Answers:**

\[ R_1 = R_a (\frac{V_{CC}}{V_{BB}}) = 150 \text{ kilo}\Omega \]
\[ R_2 = R_a (\frac{V_{CC}}{(V_{CC} - V_{BB})}) = 75 \text{ kilo}\Omega \]

(b) [9 points] Calculate the currents \( i_C, \ i_E, \) and \( i_B \).

**Answers:**

\[ -V_{BB} + i_B R_b + 0.7 = 0 \]
\[ i_B = (5 - 0.7)/[50k] = 0.086 \text{ mA} \]
\[ i_C = \beta i_B = (100)(0.086 \text{ mA}) = 8.6 \text{ mA} \]
\[ i_E = (\beta + 1) i_B = (201)(0.086 \text{ mA}) = 8.686 \text{ mA} \]

(c) [6 points] Calculate the voltages \( V_{EC} \) and \( V_o \). Note that \( R_e = 0 \text{ \Omega} \).

\[ V_a = i_C R_C = (8.6 \text{ mA})(500 \text{ Ohm}) = 4.3 \text{ V} \]
\[ V_{EC} = V_{CC} - i_C R_C = 15 - 4.3 = 10.7 \text{ V} \]

\[ V_{EC} = \quad V_o = \]
Consider the BJT circuit shown (\( v_S = 0 \)).
For the bipolar junction transistor, 
\[ \beta = 200 \text{ (nominal)}, \]
\[ V_{\text{in (turn-on)}} = 0.7 \text{ V}, \]
\[ V_{\text{CE (SAT)}} = 0.20 \text{ V}, \]
For the circuit let 
\[ V_{BB} = 3.70 \text{ V}, V_{CC} = 12.0 \text{ V}, \]
\[ R_b = 10.00 \text{ k}\Omega, R_e = 1.00 \text{ k}\Omega. \]

(a) [2 points] Identify the transistor type. (Circle one)

\[ \text{npn} \quad \text{pnp} \]
Answer: npn

(b) [3 points] What type of carrier is largest in the transistor current \( i_C \)? (Circle one)

\[ \text{holes} \quad \text{electrons} \]
Answer: electrons

(c) [15 points] Calculate the transistor currents \( i_B \), \( i_E \), and \( i_C \), and the voltages \( V_\theta \) and \( v_{CE} \). Express all answer to 3 decimal places.

Answers:
\[ -V_{BB} + i_B R_b + 0.7 + i_E R_e = 0 \quad -V_{BB} + i_B [R_b + (1 + \beta) R_e] +0.7 = 0 \]
\[ i_B = (3.7 - 0.7)/[10k + (201) 1k] = 0.0142 \text{ mA} \]
\[ i_C = \beta i_B = (200)(0.0142 \text{ mA}) = 2.844 \text{ mA} \]
\[ i_E = (\beta + 1) i_B = (201)(0.0142 \text{ mA}) = 2.858 \text{ mA} \]
\[ V_\theta = i_E R_e = (2.858 \text{ mA})(1\text{kiloOhm}) = 2.858 \text{ V} \]
\[ v_{CE} = V_{CC} - i_E R_e = 12 - 2.858 = 9.142 \text{ V} \]

\[ i_B = \quad i_E = \quad i_C = \]
\[ V_\theta = \quad v_{CE} = \]

(d) [5 points] For an unbiased bipolar transistor of the type given in this problem, sketch the equilibrium energy-band diagram. Clearly label the Fermi energy levels \( E_F \) and the intrinsic levels in each region \( E_{i,X} \). Label each region (collector, base, and emitter).
(6)

(a) [5 points] For an unbiased n-channel JFET, sketch the equilibrium energy-band diagram. Clearly label the Fermi energy level $E_F$ in each region and the intrinsic energy levels $E_{i,Gate}$ and $E_{i,Channel}$ in each region. Label each region (channel or gate).

(b) [5 points] Consider a p-channel JFET with specifications $V_{pp}$ and $I_{SS}$. For the saturation case, sketch the curve $i_{DS}$ vs. $V_{GS}$. Label key points of the curve.

(c) [6 points] Consider an n-channel JFET with specifications $V_{pp}$ and $I_{DS}$. On a single graph, sketch the current-voltage curves $i_{DS}$ vs. $V_{DS}$ for the following three input voltage cases. Clearly label which curve corresponds to each input voltage case.

A) $V_{GSS} = 0$ V
B) $-V_{pp} < V_{GSS} < 0$ V
C) $V_{GSS} < -V_{pp}$

Consider the circuit shown in which the n-channel JFET has the following specification:

$V_{pp} = 4.0$ V and $I_{SS} = 2.0$ mA.

The supply voltage is $V_{DD} = 10.0$ V. The operating current is $I_{DS} = 1.0$ mA.

i) [3 points] Assuming saturation conditions exist, calculate the needed value of $V_{i} = V_{GS}$.

Answer:

$V_{i} = V_{GS} = -1.2$ V

ii) [6 points] Calculate the resistor value $R_d$ needed to have the JFET operating voltage $V_{DS} = 6.0$ V

Answer:

$KVL\ LL\ is\ -V_{DD} + V_{DS} + i_{DS} R_d = 0$
$R_d = (V_{DD} - V_{DS})/i_{DS} = (10 - 6)/(0.001) = 4$ kiloOhm

$R_d = 4$ kiloOhm
Consider a depletion-mode MOSFET circuit with

\[ V_{PO} = 4.00 \text{ V}, \quad I_{DSS} = 2.00 \text{ mA}, \quad i_G = 0 \text{ A}, \quad \text{and} \quad V_{DD} = 15.0 \text{ V}. \]

(a) [3 points] Identify the MOSFET type (circle one).

- n-channel Depletion-mode MOSFET
- p-channel Depletion-mode MOSFET

Answer: n-channel

(b) [6 points] Determine the intercepts in symbolic terms on a \((i_{DS} \text{ vs. } v_{DS})\) graph for the load-line KVL, i.e. voltage-axis intercept when current is zero and current-axis intercept when the voltage is zero.

**Answers:**

KVL LL is \(- V_{DD} + v_{DS} + i_{DS} R_d = 0\)

- When \(i_{DS} = 0, v_{DS} = V_{DD}\)
- When \(v_{DS} = 0, i_{DS} = V_{DD}/R_d\)

**Voltage-axis intercept** = \((____, 0 \text{ A})\)  **Current-axis intercept** = \((0 \text{ V, } _____)\)

(c) [4 points] Calculate the maximum value of \(V_i\) for which the current \(i_{DS} = 0\).

**Answer:**

\[ i_{DS} = 0 \text{ for } V_i = v_{GS} = - V_{PO} = - 4 \text{ V}, \]

\[ V_i = \underline{\text{______________}} \]

(d) [8 points] If the current \(i_{DS} = 2.00 \text{ mA}\), calculate the value of \(R_d\) for which the MOSFET operates at the threshold of saturation.

**Answer:**

For operation at the threshold and \(i_{DS} = I_{DSS}, v_{GS} = 0\).

Then, \(v_{DS} - v_{GS} = v_{DS} - 0 = V_{PO} = 4 \text{ V}\)

KVL LL is \(- V_{DD} + v_{DS} + i_{DS} R_d = 0\)

- When \(R_d = (1/i_{DS})(V_{DD} - v_{DS}) = (1/0.002)(15 - 4) = 5,500 \text{ Ω}, \)

\[ R_d = \underline{\text{______________}} \]

(e) [4 points] Let \(v_{GS} = v_{GS,C}\) be the input voltage that produced \(i_{DS} = 2.00 \text{ mA}\) in part c (using the value of \(R_d\) calculated in part c). If the input voltage increases, i.e. \(v_{GS} > v_{GS,C}\), what is the condition of the MOSFET. (Circle one.)

- Saturated Operation
- Unsaturated Operation

Answer: Unsat. Operation
Consider the OpAmp circuit shown. Assume ideal OpAmp parameters.

(a) [3 points] For this ideal case, what is the input resistance of the OpAmp? (Circle one.)

\[ R_{in} = \begin{cases} 
\text{Zero Ohms} & \text{if } R_{in} = 0 \\
\text{100 Ohms} & \text{if } R_{in} = 100 \\
\text{100,000 Ohms} & \text{if } R_{in} = 100,000 \\
\text{Infinite Ohms} & \text{if } R_{in} = \infty 
\end{cases} \]

Answer: \( R_{in} = \text{Infinite Ohms} \)

(b) [3 points] What type of OpAmp circuit is shown? (Circle one.)

Inverting (\( V_o / V_{TH} \) neg.)  Non-Inverting (\( V_o / V_{TH} \) pos.)  Answer: Non-Invert

(c) [12 points] Derive the symbolic expression for the circuit gain \( V_o / V_{TH} \) in terms of the resistances.

Answer:
By KVL (no current into ideal OpAmp) \( V_o = V_{in} = V_{TH} \)
By voltage division (no current into ideal OpAmp) \( V_o = \left[ \frac{R_a}{R_a + R_b} \right] V_{in} \)
\[ V_o = \left[ \frac{R_a + R_b}{R_a} \right] V_{TH} \] (non-inverting case)
\[ V_o / V_{TH} = \left[ \frac{R_a + R_b}{R_a} \right] \quad \text{or} \quad 1 + \left( \frac{R_b}{R_a} \right) \] (non-inverting case)

\[ V_o / V_{TH} = \boxed{\text{_________}} \]

(d) [7 points] The circuit values are
\( R_{TH} = 5.00 \text{ k}\Omega, \quad R_a = (1 - x) \text{ 4.00 k}\Omega, \quad \text{and} \quad R_b = x \text{ 4.00 k}\Omega. \)
Let the voltage across \( R_A \) be \( V_A \). If \( |V_o/V_A| = 6 \), calculate the needed value of the fraction \( x \).

Answer:
By voltage division (no current into ideal OpAmp) \( V_A = \left[ \frac{R_a}{R_a + R_b} \right] V_o \)
\[ V_o = \left[ \frac{R_a + R_b}{R_a} \right] V_A \] (non-inverting case)
\[ V_o / V_A = \left[ \frac{R_a + R_b}{R_a} \right] = 1 + \left( \frac{R_b}{R_a} \right) = 1 + \left( \frac{x}{1 - x} \right) \]
\[ V_o / V_A = \left( 1 - x \right) / \left( 1 - x \right) + x / \left( 1 - x \right) = \frac{1}{1 - x} = 6 \]
x = \boxed{5/6}

\[ x = \boxed{\text{______}} \]
(9) Consider the OpAmp circuit shown.

(a) [5 points] Sketch the equivalent circuit in which the OpAmp is replaced with appropriate circuit elements. Assume ideal values for the OpAmp resistances.

(b) [10 points] Use superposition to derive the influence of just the input \( v_{sb} \) with \( v_{sa} = 0 \), i.e. \( V_{0,b} \) as a function of the resistances \( R_1 \) and \( R_2 \) and voltage \( v_{sb} \). Assume an ideal OpAmp.

Answer (\( v_{sa} = 0 \) gives a short)

By Voltage Division

\[
V_0 = \frac{R_1}{R_1 + R_2} V_{0,b}
\]

Negative Feedback produces

\[
v_{sb} = V_+ = V_0 = \frac{R_1}{R_1 + R_2} V_{0,b}
\]

\[
V_{0,b} = [1 + \frac{R_2}{R_1}] v_{sb}
\]

\[V_{0,b} = \quad \]

(c) [10 points] The resistance values are \( R_1 = 2.00 \, k\Omega \) and \( R_2 = 10.00 \, k\Omega \.

If the input voltage \( v_{sb} = 6 \, V \), calculate the input voltage \( v_{sa} \) which gives zero for the (total) output voltage, i.e. \( V_0 = 0 \). Assume an ideal OpAmp.

Answer

Negative Feedback gives \( v_{sb} = V_+ = V_0 = 6 \, V \).

KCL at “+” gives

\[
\frac{v_{sb} - v_{sa}}{R_1} + \frac{v_{sb} - V_0}{R_2} = 0
\]

\[
6 - v_{sa} = \frac{R_1}{R_1} + \frac{6}{R_2} = 0
\]

\[
v_{sa} = R_1 \left( \frac{6}{R_1} \right) + \frac{6}{R_2} = (2000) \left( \frac{6}{2000} \right) + (6/10000) = 7.20 \, V
\]

Alternative

Superposition Non-inverting

\[
V_{0,b} = \left[ 1 + \frac{R_2}{R_1} \right] v_{sb} = \left[ 1 + \frac{10000}{2000} \right] 6 = 36 \, V
\]

Superposition Inverting

\[
V_{0,a} = - \left( \frac{R_2}{R_1} \right) v_{sa} = - \frac{10000}{2000} v_{sa} = -5 v_{sa}
\]

Hence, \( V = 0 \) for \( V_{0,a} = -5 v_{sa} = -36 \, V \)

\[
v_{sa} = 36/5 = 7.20 \, V
\]

\[v_{sa} = \quad \]
Consider crystalline Si which has a bandgap energy of $E_G = 1.12$ eV. Complete the table with the correct calculation and selection, i.e. calculate the photon energies of these wavelengths and note whether Si is absorbing or transparent.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Photon Energy in eV</th>
<th>Si is (circle one)</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td></td>
<td>Absorbing</td>
</tr>
<tr>
<td>1500</td>
<td></td>
<td>Absorbing</td>
</tr>
</tbody>
</table>

**Answer**

\[
E_p = \frac{hc}{\lambda} = \frac{(4.136 \times 10^{-15} \text{ eV-s}) (2.998 \times 10^{10} \text{ cm/s})}{(0.9 \times 10^{-4} \text{ cm})} = 1.38 \text{ eV}
\]

Absorbing since $E_p > E_G$

\[
E_p = \frac{hc}{\lambda} = \frac{(4.136 \times 10^{-15} \text{ eV-s}) (2.998 \times 10^{10} \text{ cm/s})}{(1.5 \times 10^{-4} \text{ cm})} = 0.827 \text{ eV}
\]

Transparent since $E_p < E_G$

Consider the Si pin photodiode circuit shown with source voltage $V_S = -12.0 \text{ V}$ and with load resistance $R = 5000 \text{ Ohms}$. The reverse saturation current (for “dark” conditions) is 0.20 mA. Assume the photodiode voltage $|V| \gg kT/q$ and room temperature.

**Answers:**

For Dark Conditions, $I = -I_0 = -0.20 \text{ mA}$

\[
\text{KVL LL is } -V_S + V + IR = 0
\]

\[
V = V_S - IR = (-12) - (-0.20 \text{ mA})(5000 \text{ Ohms}) = -11.0 \text{ V}
\]

\[
V = \text{-----------------} \quad I = \text{-----------------}
\]

For these conditions, $I = -I_0 - I_{\text{light}} = -0.20 - 1.0 \text{ mA} = -1.2 \text{ mA}$

\[
\text{KVL LL is } -V_S + V + IR = 0
\]

\[
V = V_S - IR = (-12) - (-1.20 \text{ mA})(5000 \text{ Ohms}) = -6.0 \text{ V}
\]

\[
V = \text{-----------------} \quad I = \text{-----------------}
\]