Electrical Engineering Advancement Exam III

FALL SEMESTER 2020

CLOSED BOOK, CLOSED NOTES
2 HOUR TIME LIMIT

CALCULATORS ARE ALLOWED
(calculators without communication capability only)

ELECTRONIC DEVICES WITH COMMUNICATION CAPABILITY
(electronic devices such as cell phone, pagers, and iPads)

MAY NOT BE USED DURING THE EXAMINATION
(If such devices ring or are visible, a 10% penalty will be given for the first occurrence and exam failure for the second.)

There are 10 problems: please look over the exam to make sure that you have 10 different problems. **Do any eight (8) problems!** Draw a large X through the two problems that you do not want to be graded. If you do not indicate which problems you want to leave out, the first 8 problems will be graded.

Do all work for each problem only on the page supplied for that problem (you may use both sides). **DO NOT**, for instance, continue Problem #3 on the back of Problem #2. Extra blank paper will be supplied if needed. If extra paper is used, show the additional work for each problem on a separate sheet and staple the extra sheet(s) to the appropriate problems.
(a) [5 points] For intrinsic semiconductors at equilibrium, the electron concentration (number of electrons per unit volume) at room temperature is ???  Circle the one correct choice.

- Equal to 1.0 x 10^{22} \text{ cm}^{-3}
- Equal to the hole concentration
- Equal to 4 electron per atom times the number of atoms per \text{ cm}^{3}
- None of the above are true

Answer: Equal to the hole concentration

(b) [5 points] Because the energy-gap $E_G$ of crystalline silicon is bigger than that of crystalline germanium, the electron concentration of intrinsic silicon has what relationship to the electron concentration of intrinsic germanium at room temperature?  Circle the one correct choice.

- $n_{\text{Si}} > n_{\text{Ge}}$
- $n_{\text{Si}} < n_{\text{Ge}}$
- Undetermined for given information

Answer: $n_{\text{Si}} < n_{\text{Ge}}$

(c) [5 points] In a pn step junction diode, the electric field inside the depletion region points in which direction (i.e. is positive for which direction)?

- From the n region to the p region
- From the p region to the n region
- Direction depends on bias voltage

Answer: From the n region to the p region

(d) [10 points] Sketch the energy band diagram for a step pn junction for forward bias conditions. Label the edge of the conduction band and the valence band, the Fermi level in both n and p regions, the difference $qV_n$. 

Problem Score 25/25
An abrupt silicon p-n junction has an acceptor concentration $10^{16}$ cm$^{-3}$ (no donors) in the p-side and a donor concentration $10^{16}$ cm$^{-3}$ (no acceptors) in the n-side, respectively. The intrinsic carrier concentration of silicon is $1.50 \times 10^{10}$ cm$^{-3}$ at room temperature (300 K). Answer the following questions assuming all dopants are ionized at room temperature.

(a) [7 points] Calculate the potential difference across the junction at equilibrium (i.e. contact potential $V_o$).

$$V_o = (kT/q) \ln \left[ \frac{N_{AP,Effective} N_{DN,Effective}}{n_i^2} \right] = (0.0259 \text{ V}) \ln[(10^{16} \times 10^{16}) / (1.5 \times 10^{10})^2]$$

$$V_o = 0.695 \text{ V}$$

(b) [6 points] Calculate the ratio of p-side depletion width $x_{po}$ versus n-side depletion width $x_{no}$ at equilibrium, i.e. $x_{po}/x_{no}$.

$$N_{AP,Effective} x_{po} = N_{DN,Effective} x_{no}$$

$$x_{po} / x_{no} = N_{DN,Effective} / N_{AP,Effective} = 1$$

(c) [6 points] Calculate the total potential difference in eV across the junction when a forward bias voltage of $V_A = +0.30$ V is applied, i.e. determine the difference between the band edge energy on the p-side of the junction with respect to the n-side.

$$qV_o - qV_A = 0.695 - 0.30 = 0.395 \text{ V}$$

(d) [6 points] Calculate the ratio of total depletion width at equilibrium versus total depletion width with a forward bias of $V_A = +0.20$ V, i.e. $W_o/W$.

$$W_o / W = \sqrt{[V_o/(V_o - V_A)]} = \sqrt{(0.695)/(0.395)} = 1.326$$

$$W_o / W = \text{___________________}$$
Consider an idealized diode with a reverse saturation current 0.01 mA and an abrupt turn-on voltage of 0.70 V (assume no breakdown and operation away from the knee of the diode curve). For the circuit shown, both resistors are 100 Ω, the constant source is \( V_P = 5.0 \) V, and the time-dependent source is \( V_S = 20 \sin(\omega t) \) V.

(a) [12 Points] State the bias condition of the diode. Calculate the peak positive voltage \( V \) and the associated current \( I_d \) through the diode. Calculate the answers to at least four significant digits (x.xxxx).

KVL gives \( V = - V_P - V_d = - 5 - V_d \).
For \( V_S = + 20 \) V, the diode is reverse biased. Hence, \( I_d = - 0.01 \) mA.

KCL at the node gives
\[
\frac{(V - (+20))}{(100 \, \Omega)} - (-0.00001 \, \text{A}) + \frac{(V)}{(100 \, \Omega)} = 0
\]
\( V = \frac{(1/2) [+20 + (-0.001)]}{=} + 9.9995 \, \text{V} \)

(b) [13 Points] State the bias condition of the diode. Calculate the peak negative voltage \( V \) and the associated current \( I_d \) through the diode.

KVL gives \( V = - V_P - V_d = - 7 - V_d \).
For \( V_S = - 20 \) V, the diode is forward biased. Hence, \( V_d = 0.7 \) V.

Hence, the resistor voltage is at the limit with \( V = - 5 - 0.7 = - 5.7 \) V.

KCL at the node gives
\[
\frac{[-(5.7 \, \text{V}) - (-20 \, \text{V})]}{(100 \, \Omega)} - I_d + \frac{(-5.7 \, \text{V})}{(100 \, \Omega)} = 0
\]
\( I_d = \frac{[2(-5.7) - (-20 \, \text{V})]}{(100 \, \Omega)} = + 0.0860 \, \text{A} \)

Sketch (not required in solution) +9.9995 V

\( V = \frac{=}{} \)
\( I_d = \frac{=}{} \)
Consider the npn-transistor Darlington amplifier circuit shown with three npn BJTs. The transistors are identical with a nominal beta \( \beta = \beta_1 = \beta_2 = \beta_3 = 10 \), a turn-on voltage \( V_{th} = 0.70 \) V, and a saturation emitter-collector voltage \( V_{EC(SAT)} = 0.20 \) V. The circuit values are \( V_{CC} = 100 \) V, \( V_{BB} = 7.1 \) V, \( R_{B1} = 10.0 \) k\( \Omega \), and \( R_C = 10 \) \( \Omega \).

(a) [18 points] Calculate the currents \( i_{C1}, i_{C2}, \) and \( i_{C3} \).

\[
\text{KVL Base Side: } -V_{BB} + 3V_{th} + i_{B1} R_{B1} = 0
\]
\[
i_{B1} = \frac{1}{R_{B1}} (V_{BB} - 3V_{th}) = \frac{1}{10000} (7.1 - 2.1) = 0.50 \text{ mA}
\]
\[
i_{C1} = \beta i_{B1} = (10) (0.50 \text{ mA}) = 5.0 \text{ mA}
\]
\[
i_{C2} = \beta i_{B2} = \beta i_{C1} = \beta (\beta +1) i_{B1} = (10)(11) (0.50 \text{ mA}) = 55.0 \text{ mA}
\]
\[
i_{C3} = \beta i_{B3} = \beta i_{C2} = \beta (\beta +1) i_{B2} = \beta (\beta +1) i_{C1} = \beta (\beta +1) (\beta +1) i_{B1}
\]
\[
i_{C3} = (10)(11)(11) (0.50 \text{ mA}) = 605.0 \text{ mA}
\]

\[i_{C1} = \quad i_{C2} = \quad i_{C3} = \quad\]

(b) [7 points] Calculate the total current through the resistor \( R_C \) and the voltage across \( R_C \), i.e. \( i_{C\text{Total}} \) and \( V_o \).

\[
i_{C\text{Total}} = i_{C1} + i_{C2} + i_{C3} = 5 \text{ mA} + 55 \text{ mA} + 605 \text{ mA} = 665 \text{ mA}
\]

By Ohm’s Law: \( V_o = i_{C\text{Total}} R_{C\text{Total}} = (665\text{mA}) (0.010 \text{ k}\Omega) = 6.65 \text{ V}\)

\[i_{C\text{Total}} = \quad V_o = \quad\]
Consider the pnp BJT circuit shown with 
\[ \beta = 99, \ V_{io} = 0.7 \text{ V}, \]
\[ V_{BB} = 5.0 \text{ V}, \ V_{CC} = 12.0 \text{ V}, \]
\[ R_B = 12.00 \text{ k}\Omega, \ R_E = 100 \text{ } \Omega, \]
and \[ R_C = 100 \text{ } \Omega. \]
The operating transistor voltage \( v_{CE} = 12.0 \text{ V}. \)
Assume that the circuit is operating in the active region.

(a) [12 points] Calculate the transistor currents \( i_B \) and \( i_E \).

Answers:
KVL Base Side:
- \( - V_{BB} + V_{io} + i_B R_B + i_E R_E = 0 \)
- \( - V_{BB} + V_{io} + i_B R_B + i_B (1 + \beta) R_E = 0 \)

\[ i_B = \frac{(V_{BB} - 3 V_{io})}{(1 + \beta) R_E + R_B} = \frac{(5.0 - 0.7)}{(1 + 99) 100 + 12000} = 0.1955 \text{ mA} \]

\[ i_E = \beta i_B = (1 + 99) (0.1955 \text{ mA}) = 19.55 \text{ mA} \]

KVL LL gives
- \( - V_{CC} + v_{CE} + i_E R_E = 0 \)

\[ i_E = \frac{1}{R_E}(V_{CC} - v_{CE}) = \frac{1}{1000}(21 - 12) \]

\[ i_E = 9 \text{ mA} \]

\[ i_n = [1/(\beta + 1)] i_E = (1/200)(0.009) = 0.000045 \text{ A} = 0.045 \text{ mA} \]

\[ i_c = [\beta/(\beta + 1)] i_E = (199/200)(0.009) = 0.008955 \text{ A} = 8.955 \text{ mA} \]

\[ i_B = \frac{1}{(\beta + 1)} i_E = \frac{1}{200}(0.009) = 0.000045 \text{ A} = 0.045 \text{ mA} \]

(b) [13 points] Calculate the operating point \( i_C \) and \( v_{EC} \).

Answer:
\[ i_C = \beta i_n = (99) (0.1955 \text{ mA}) = 19.35 \text{ mA} \]

Load-Line Collector-side KVL
- \( - V_{CC} + i_E R_E + i_C R_C + v_{EC} = 0 \)

\[ v_{EC} = V_{CC} - i_E R_E - i_C R_C = V_{CC} - i_C \left[ \frac{1}{(\beta + 1)} R_E \right] - \frac{i_C}{R_C} \]

\[ v_{EC} = (12) - (19.55 \text{ mA})(100/99)(100) + 100 = 8.11 \text{ V} \]

\[ i_C = \frac{1}{(\beta + 1)} i_E = \frac{1}{200}(0.009) = 0.000045 \text{ A} = 0.045 \text{ mA} \]

\[ v_{EC} = \frac{1}{(\beta + 1)} v_{CE} = \frac{1}{200}(8.11) = 0.04055 \text{ V} \]
(a) [5 points] For the inverter MOSFET circuit shown, the enhancement-mode p-channel MOSFET has parameters $V_{on}$ and $K$ and the depletion-mode p-channel MOSFET has parameters $V_{po}$ and $I_{DS}$. For the upper or load MOSFET, sketch the IV characteristic $i_{SD2}$ vs. $v_{SD2}$. State the control voltage $v_{SG2}$ value and label the axes and any key voltage or current values.

(b) [5 points] For the inverter MOSFET circuit shown in part a, the enhancement-mode p-channel MOSFET has parameters $V_{on}$ and $K$ and the depletion-mode p-channel MOSFET has parameters $V_{po}$ and $I_{DS}$. Sketch the load line for the lower or driver transistor $i_{SD1}$ vs. $v_{SD1}$. Label the axes and important intercept values.

(c) [9 points] Consider an n-channel MOSFET circuit with $V_{on} = 3.00$ V, $K = 1.00$ mA/V$^2$, $R_d = 1.00$ kΩ, $V_{DD} = 20.0$ V, and $V_i = 5.0$ V. Calculate the operating point or Q-point, i.e. $i_{DS}$ and $v_{DS}$.

Note that $v_{GS} = V_i$.
Assume saturation.

$$i_{DS} = KV_{on}^2[(v_{CS}/V_{on}) - 1]^2 = (1.0 \text{ mA/V}^2) (3)^2 [(5/3) - 1]^2 = 4.0 \text{ mA}$$

The KVL load line gives $v_{DS} = V_{DD} - i_{DS} R_d = 20 - (0.004) (1000) = 16$ V
Check: $v_{DS} - v_{GS} > - V_{on}$ or 16 - 5 = 11 $\geq$ - 3. Saturation confirmed

Answers: $v_{DS} = 16$ V and $i_{DS} = 4.0$ mA

$$v_{DS} = \text{_________} \quad i_{DS} = \text{_________}$$

(d) [6 points] For the MOSFET circuit in part b with $V_{on} = 3.00$ V, $K = 1.00$ mA/V$^2$, $V_{DD} = 20.0$ V, and $V_i = 5.0$ V, calculate the range of $R_d$ for which the transistor operation stays in saturation.

Saturation must satisfy $v_{DS} - v_{GS} > - V_{on}$ or $v_{DS} > + v_{GS} - V_{on}$
The threshold of saturation occurs for $v_{DS} = V_{GS} - V_{on}$ or $v_{DS} = 5 - 3 = 2$ V
The saturation current for $V_i = 5.0$ V is

$$i_{DS} = K(V_{on})^2[(v_{GS}/V_{on}) - 1]^2 = (1.0 \text{ mA/V}^2) (3)^2 [(5/3) - 1]^2 = 4.0 \text{ mA}$$

By KVL, the threshold value for resistance is

$$- V_{DD} + v_{DS} + i_{DS} R_d = 0 \quad \text{or} \quad R_d = (+ V_{DD} - v_{DS}) / i_{DS} = (20 - 2) / (0.004) = 4.50 \text{ kΩ}$$

The range is then

$$R_d \leq 4.50 \text{ kΩ}$$

Range of $R_d$: \_________
(a) [5 points] Consider a p-channel JFET with specifications $V_{po}$ and $I_{DS}$.
Sketch the saturation curve $i_{SD}$ vs. $v_{SG}$ i.e. the curve for saturation conditions. Label all intercepts.

(b) [20 points] Consider the p-channel JFET circuit shown. The transistor parameters/specifications are $V_{po} = 4.00$ V and $I_{DS} = 10.0$ mA. The circuit values are $R_d = 1.70$ kΩ and $V_{DD} = 20.0$ V. Calculate the input voltage $v_{SG}$; the operating point or Q-point, i.e. $i_{SD}$ and $v_{SD}$; and the load-line intercepts for current and voltage. Sketch the current-voltage characteristic and the load-line on the same graph. Label the operating point (Q-point) and the load-line intercepts.

Note that $v_{SG} = 0$
Assume saturation.

$$i_{SD} = I_{DS}[1 + (v_{SD}/V_{po})]^2 = (10.0 \text{ mA})[1 + 0]^2 = 10.0 \text{ mA}$$
The KVL load line gives $v_{SD} = V_{DD} - i_{SD}R_d = 20 - (0.010)(1700) = 3 \text{ V}$
Check: Is $v_{SD} - v_{SG} \geq V_{po}$? $3 - 0 = 3 < 4$. Unsaturated Conditions

Simultaneous solution needed for LL and $i_{SD}$ unsaturated equation

$$i_{SD} = I_{DS}[2(1 + (v_{SD}/V_{po}))^2 (v_{SD}/V_{po}) - (v_{SD}/V_{po})^2] = (V_{DD} - v_{SD})/R_d$$
$$0.010[2(1 + 0)(v_{SD}/4)^2 - (v_{SD}/4)^2] = (20 - v_{SD})/1700$$
$$1.0625(v_{SD})^2 - 9.5(v_{SD}) + 20 = 0$$
gives $v_{SD} = 9.5488 \text{ V, 3.3923 V}$

$v_{SD} = 3.3923 \text{ V, 3.3923 V}$ is in the correct range
Then, $i_{SD} = (V_{DD} - v_{SD})/R_d = (20 - 3.3923)/1700 = 9.769 \text{ mA}$

Answers: $v_{SD} = 3.3923 \text{ V}$ and $i_{SD} = 9.769 \text{ mA}$

$v_{SG} = \underline{\text{______}}$  $v_{SD} = \underline{\text{______}}$  $i_{SD} = \underline{\text{______}}$

Voltage-axis intercept = (______, 0 A)  Current-axis intercept = (0 V, ____ )
(8)

(a) [8 points] The OpAmp shown has ideal resistances \( R_{in} \) and \( R_{out} \) and has a finite open-loop gain \( A = 20,000 \). For the circuit shown, calculate \( V_o/V_S \). Express the answer to at least 5 significant digits (x.xxxxx).

\[
\text{KVL: } -V_S + \Delta v + V_O = 0 \quad \text{and} \quad A \Delta v = V_O
\]

\[
-V_S + V_o/A + V_O = 0
\]

\[
V_O/V_S = \frac{1}{[(1/A) + 1]} = 1 / [(1/20000) + 1] = 0.99995
\]

\[
V_O/V_S = \text{__________________________}
\]

(b) [7 points] For the OpAmp circuit shown, sketch the equivalent circuit in which the OpAmp is replaced with appropriate circuit elements. Assume non-ideal OpAmp parameters.

(c) [3 points] What type of OpAmp circuit is shown in part b? (Circle the best choice.)
   
   Inverting \( (V_O / V_{TH} \text{ neg.}) \)  \hspace{1cm} Non-Inverting \( (V_O / V_{TH} \text{ pos.}) \)  \hspace{1cm} Answer: Invert.

(d) [7 points] Use the OpAmp circuit shown in part b and assume ideal parameters. Consider the OpAmp circuit shown. The OpAmp is ideal. The Thevenin circuit values are

\[
V_{TH} = 5.00 \text{ V and } R_{TH} = 1.00 \text{ k}\Omega.
\]

The feedback resistor \( R_2 = 20.00 \text{ k}\Omega \). Calculate the value of resistor \( R_1 \) such that the magnitude of the output voltage to the Thevenin voltage is \( |V_o/V_{TH}| = 5 \).

\[
\text{KCL for the (-) terminal:}
\]

\[
(0 - v_{TH})/(R_1 + R_{TH}) + (0 - v_o)/R_2 = 0
\]

Hence,

\[
V_o/V_{TH} = \frac{R_2}{(R_1 + R_{TH})} \]

\[
|V_o/V_{TH}| = 5 \quad \Rightarrow \quad |R_2/(R_1 + R_{TH})| = 20000/(R_1 + 1000)
\]

\[
R_1 = \text{__________________________}
\]

Problem Score /25
(9)

(a) [8 points] Identify the two OpAmp circuits shown. (Select the best answers.)

Circuit A: \[ \text{Differentiating B and Integrating A} \]

Circuit _______ is a Differentiating circuit \( (V_o = -RC \frac{d(V_s)}{dt}) \)

Circuit _______ is a Integrating \( (V_o = (-1/RC) \int V_s \, dt) \)

(b) [8 points] Consider the OpAmp circuit shown. Assume an ideal OpAmp. The resistance values are

\[ R_1 = 1.00 \, \text{k}\Omega, \quad R_2 = 5.00 \, \text{k}\Omega, \]

\[ R_3 = 2.00 \, \text{k}\Omega, \quad R_4 = 2.00 \, \text{k}\Omega. \]

Operation is in the linear range.

Calculate the output \( V_o \) as a function of \( v_1 \) when \( v_2 = 0 \). Hint: Use superposition.

No current in \( R_3 \) and \( R_4 \) so \( V_+ = 0 \). \( \Delta v = 0 \) due to negative feedback.

KCL at (-) terminal with \( V_- = 0 \) gives\( (0 - v_1)/(R_3) + (0 - V_o)/R_2 = 0 \)

\[ V_o = \frac{R_2}{R_1} v_1 \quad \text{(inverting case)} \]

(c) [9 points] Consider the same OpAmp circuit. Assume an ideal OpAmp. The resistance values are the same as in (b) and are

\[ R_1 = 1.00 \, \text{k}\Omega, \quad R_2 = 5.00 \, \text{k}\Omega, \]

\[ R_3 = 2.00 \, \text{k}\Omega, \quad R_4 = 2.00 \, \text{k}\Omega. \]

Operation is in the linear range.

Calculate the output \( V_o \) as a function of \( v_2 \) when \( v_1 = 0 \). Hint: Use superposition.

By voltage division (no current into ideal OpAmp) \( V_+ = [(R_4)/(R_3 + R_4)] v_2 \)

\( V_- = V_o \) since \( \Delta v = 0 \) (negative feedback).

By voltage division (no current into ideal OpAmp) \( V_+ = [(R_1)/(R_1 + R_2)] V_o \)

\[ V_o = \frac{[(R_1 + R_2)/R_1] V_-}{[(R_1 + R_2)/R_2]} = \frac{[(R_1 + R_2)/R_1] [(R_3 + R_4)]}{[(R_3 + R_4)] v_2} \]

\[ V_o |_{v_1=0} = V_{o2} = (__________________________) v_2 \]
Consider the Si pin photodiode circuit shown with source voltage $V_S$ and resistance $R = 6000$ Ohms. The bandgap energy for Si is $E_G = 1.12$ eV. The reverse saturation current (for “dark” conditions) is 0.10 mA. Light of wavelength $\lambda = 0.900 \mu m$ is incident; the efficiency is 0.90. Assume the photodiode voltage $|V| > kT/q$ and room temperature.

(a) [5 points] For photoconductive operation as described in the lecture notes, what is the correct bias condition for the photodiode? Choose (circle) the correct choice.

- Forward bias
- No Bias
- Reverse Bias

Answer: Reverse Bias.

(b) [5 points] Calculate the photon energy $E_P$ for this wavelength. Express the photon energy in eV.

Answer:

$$E_P = \frac{hc}{\lambda} = \frac{4.136 \times 10^{-15} \text{ eV-s}}{(2.998 \times 10^{10} \text{ cm/s})/(0.9 \times 10^{-4} \text{ cm})} = 1.378 \text{ eV}$$

(c) [10 points] For an absorbed optical power $P$ at this wavelength $\lambda = 0.900 \mu m$, the photo-induced current is $I_{\text{light}} = 1.0$ mA. Calculate the absorbed optical power in W.

Answer:

$$P = \frac{I_{\text{light}} \cdot (hc)}{(\eta q \cdot \lambda)} = \frac{(0.001 \text{ C/s}) \cdot (6.626 \times 10^{-34} \text{ J-s})(2.998 \times 10^{10} \text{ cm/s})}{(0.9)(1.602 \times 10^{-19} \text{ C})(0.00009 \text{ cm})} = 1.53 \text{ mW}$$

(d) [5 points] For the photo-induced current of $I_{\text{light}} = 1.0$ mA, the operating voltage is $V = -7.50$ V. Calculate the needed source voltage $V_S$.

Answer:

$$I = -I_L - I_{\text{light}} = -(0.1 \text{ mA}) - 1.0 \text{ mA} = -1.1 \text{ mA}$$

KVL LL gives

$$-V_S + V + I \cdot R = 0$$

$$V_S = V + I \cdot R = -(7.5 \text{ V}) + (1.1 \text{ mA})(6000 \text{ Ohms}) = -14.1 \text{ V}$$

$V_S = -$