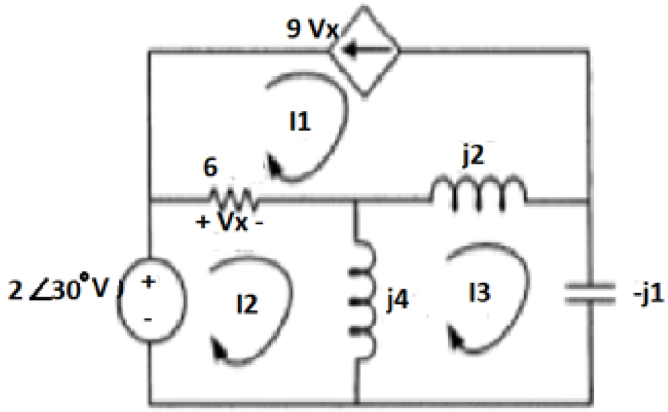
(1) a. Write a set of mesh current equations for the circuit to find I_1, I_2 and I_3 in matrix form. You must eliminate the control variable from your equations. (15)

b. Solve for phasor mesh current I_2. (5)

c. Express steady state mesh current $i_2(t)$ in time domain if ω=1250 rad/s. (5)



a. M1: $\tilde{I}_1 = -9\tilde{V}_x$; $\tilde{V}_x = 6(\tilde{I}_2 - \tilde{I}_1)$

$\Rightarrow \tilde{I}_1 = -54(\tilde{I}_2 - \tilde{I}_1)$

$\Rightarrow 53\tilde{I}_1 - 54\tilde{I}_2 = 0$ — 1

M2: $-2\angle 30^\circ + 6(\tilde{I}_2 - \tilde{I}_1) + j4(\tilde{I}_2 - \tilde{I}_3) = 0$

$-6\tilde{I}_1 + (6+j4)\tilde{I}_2 + (-j4)\tilde{I}_3 = 2\angle 30^\circ$ — 2

M3: $j4(\tilde{I}_3 - \tilde{I}_2) + j2(\tilde{I}_3 - \tilde{I}_1) - j1(\tilde{I}_3) = 0$

$(-j2)\tilde{I}_1 + (-j4)\tilde{I}_2 + (j5)\tilde{I}_3 = 0$ — 3

$$\begin{bmatrix} 53 & -54 & 0 \\ -6 & (6+j4) & -j4 \\ -j2 & -j4 & j5 \end{bmatrix} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2\angle 30^\circ \\ 0 \end{bmatrix}$$

b) $\tilde{I}_1 = 2.43\angle 127.77^\circ$ A

$$\boxed{\tilde{I}_2 = 2.387\angle 127.77^\circ \text{ A}}$$

$\tilde{I}_3 = 2.88\angle 127.77^\circ$ A

c) $i2(t) = 2.39\cos(1250t + 127.77^\circ)$ A

Simultaneous equation solution if not using calculator

$$(1) \Rightarrow \tilde{I}_1 = \frac{54}{53}\tilde{I}_2$$

$$(3) \Rightarrow \tilde{I}_3 = \frac{(j4)\tilde{I}_2 + (j2)\tilde{I}_1}{(j5)}$$

$$= \frac{4}{5}\tilde{I}_2 + \frac{2}{5}\tilde{I}_1$$

$$= \frac{4}{5}\tilde{I}_2 + \frac{2}{5}\left(\frac{54}{53}\right)\tilde{I}_2$$

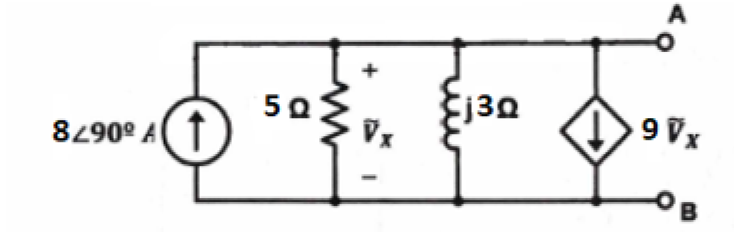
$$\tilde{I}_3 = \frac{64}{53}\tilde{I}_2$$

$$(2) \Rightarrow -6\left(\frac{54}{53}\right)\tilde{I}_2 + (6+j4)\tilde{I}_2 + (-j4)\frac{64}{53}\tilde{I}_2 = 2\angle 30^\circ$$

$$(0.838\angle -97.77^\circ)\tilde{I}_2 = (2\angle 30^\circ)$$

$$\Rightarrow \tilde{I}_2 = \frac{2\angle 30^\circ}{0.838\angle -97.77^\circ} = \underline{2.387\angle 127.77^\circ \text{ A}}$$

(2) Find and sketch the Thevenin Equivalent Circuit with respect to terminals A and B. Calculate the Thevenin voltage and impedance. (25 points)



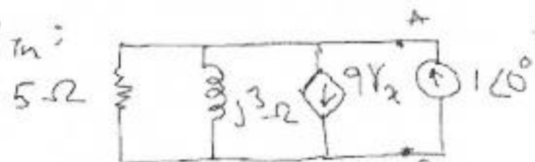
$\tilde{V}_{AB} = \tilde{V}_x$ KCL at top node:

$$-8\angle 90^\circ + \frac{\tilde{V}_{AB}}{5} + \frac{\tilde{V}_{AB}}{j3} + 9\tilde{V}_{AB} = 0$$

$$\tilde{V}_{AB}\left(\frac{1}{5} + \frac{1}{j3} + 9\right) = 8\angle 90^\circ \Rightarrow \tilde{V}_{AB} = \frac{8\angle 90^\circ}{9.21\angle 2.07}$$

$$\boxed{\tilde{V}_{AB} = 0.869\angle 92.07\ \text{V}}$$

Z_{Th}:

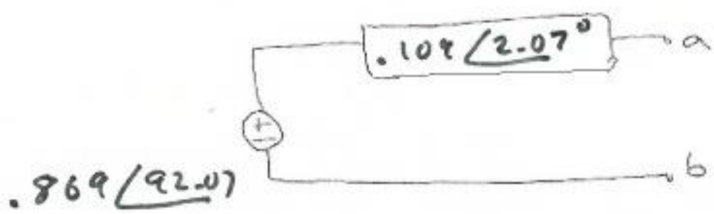


$$\boxed{Z_{Th} = Z_{AB} = \frac{\tilde{V}_{AB}}{1\angle 0^\circ} \text{ (Ohm's Law)}}$$

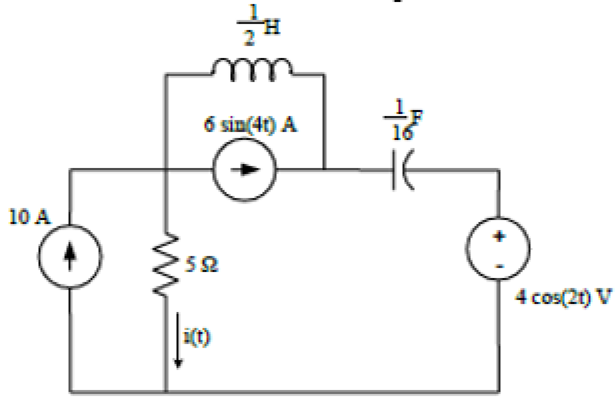
KCL $\frac{\tilde{V}_{AB}}{5} + \frac{\tilde{V}_{AB}}{j3} + 9\tilde{V}_{AB} = 1\angle 0^\circ$

$$\tilde{V}_{AB} = \frac{1\angle 0^\circ}{9.21\angle 2.07} = .109\angle 2.07^\circ$$

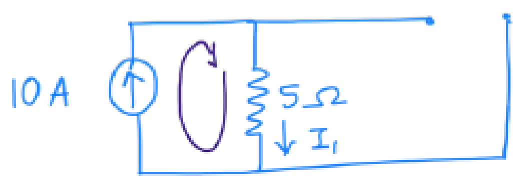
$$Z_{Th} = \frac{.109\angle 2.07^\circ}{1\angle 0^\circ}\ \Omega \quad \text{or } 0.1089 + j0.0039\ \Omega$$



(3) For the following circuit, determine the time domain current, i(t) using Superposition. Show/draw the circuit used for each part of the solution. (25)



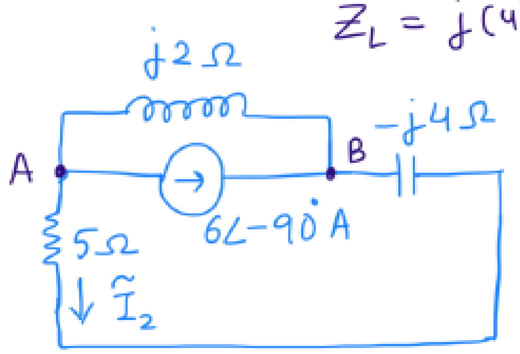
• $\underline{DC \Rightarrow \omega = 0}$ $\quad Z_C \to \infty, \; Z_L = 0$



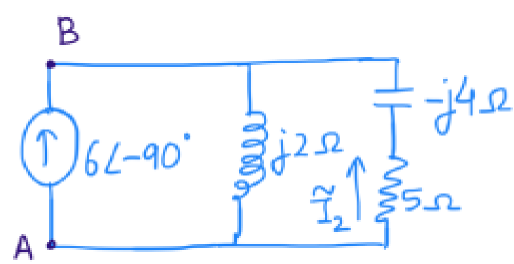
$I_1 = 10\,A$ — ①

• $\underline{\omega = 4\ rad/s}$ $\quad Z_C = \left[j(4)\left(\frac{1}{16}\right)\right]^{-1} = -j4\,\Omega$

$Z_L = j(4)\left(\frac{1}{2}\right) = j2\,\Omega$



⇒



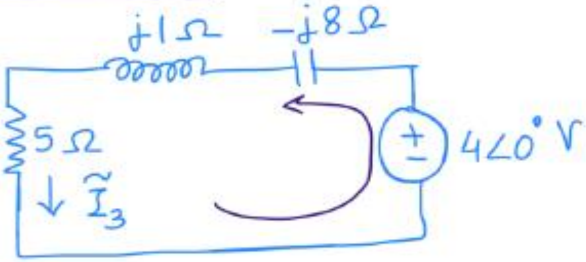
$$\tilde{I}_2 = -(6\angle -90^\circ)\left[\frac{j2 \,//\, (5-j4)}{(5-j4)}\right]$$

$$= (6\angle 90^\circ)\left(\frac{0.69 + j2.28}{5-j4}\right)$$

$$\tilde{I}_2 = 2.23\angle -158.199^\circ \text{ A} \quad \underline{\text{or}} \quad -2.23\angle 21.8^\circ \text{ A}$$

$$i_2(t) = 2.23 \cos(4t - 158.199^\circ) \text{ A} \quad — ②$$

- $\underline{\omega = 2 \text{ rad/s}}$



$$\tilde{I}_3 = \tilde{I}_{in} = \frac{4\angle 0^\circ}{5 - j7}$$

$$\tilde{I}_3 = 0.465\angle 54.46^\circ \text{ A}$$

$$i_3(t) = 0.465 \cos(2t + 54.46^\circ) \text{ A} \quad — ③$$

$$①, ②, ③ \Rightarrow \quad i(t) = I_1 + i_2(t) + i_3(t)$$

$$i(t) = \underline{10 + 2.23 \cos(4t - 158.199^\circ) + 0.465 \cos(2t + 54.46^\circ) \text{ A}}$$

EE 2120 Final Exam Dec. 17, 2020

Name:____

(4) A load draws $|S| = 10{,}000$ VA from a 60-Hz sinusoidal source at a power factor of 0.707 leading. (5 points each belw)

(1) What is the average power delivered to the load?
(2) What is the reactive power delivered to the load?
(3) If $V_{rms} = 120$ volt, what is the peak current, I_m ?
(4) What is the complex power S ?
(5) What is the load impedance Z ?

(1) $P = \underbrace{|S|}_{10{,}000} \underbrace{\cos\theta}_{0.707} = 7070$ watt

(2) $Q = |S| \sin\theta = -|S|\sqrt{1-\cos^2\theta} = -7070$ VAR

$\bar{S} = P + jQ$

(3) $|\tilde{S}| = \left|\frac{1}{2}\tilde{V}\tilde{I}^*\right| = \frac{1}{2}V_m I_m \rightarrow I_m = 117.85\,A$

$|\tilde{S}| = 10{,}000$, $V_m = 120\sqrt{2}$

(4) $\bar{S} = P + jQ = \underline{7070 - j7070}$

(5) $\bar{S} = \frac{1}{2}\tilde{V}\tilde{I}^* = \frac{1}{2}I_m^2\tilde{Z}$

$\tilde{V} = \tilde{I}\tilde{Z}$

$\tilde{Z} = \frac{2\bar{S}}{I_m^2} = 2\left[\frac{7070(1-j)}{117.85^2}\right] = 2\left[0.509(1-j)\right]$

$= 1.018\,(1-j)$

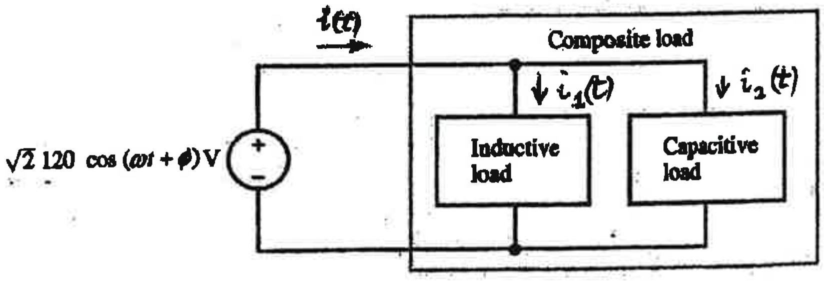
Problem #5: In the circuit below, an inductive load draw 1000 watt at "power factor", PF=0.9 lagging from a 120 V rms source. In an effort to raise the power factor seen by the source, a small capacitive load is placed in parallel with the inductive load. The capacitive load draw 10 watt at PF=0.02 leading,

(10 pts) (1) Find the complex power supplied by the source, S_s .

(5 pts) (2) Find the rms current supplied by the source, I_{rms} .

(5 pts) (3) Find the rms current supplied to the inductive load, I_{1rms} .

(5 pts) (4) Find total impedance, Z , with respect to the source.



(a) $\tilde{S}_s = \tilde{S}_1 + \tilde{S}_2$

$$\tilde{S}_1 = P_1 + jQ_1$$

$$= \underbrace{\frac{1}{2} V_m I_m \overset{0.9}{\cos\theta}}_{1000} + j \underbrace{\frac{1}{2} V_m I_m}_{\left(\frac{1000}{0.9}\right)} \underbrace{\sin\theta}_{\sqrt{1-\cos^2\theta} = \sqrt{1-(0.9)^2} = \sqrt{0.19} = 0.4358}$$

$$= 1000 + j\left(\frac{1000}{0.9}\right) \cdot 0.4358 = \underline{1000 + j\,484.3} \quad (= 1111.1 \angle 25.84°)$$

$$\tilde{S}_2 = 10 - j\left(\frac{10}{0.02}\right)\underbrace{\sqrt{1-(0.02)^2}}_{0.9998} = 10 - j\,499.9 \quad (= 500 \angle -88.85)$$

$$\tilde{S}_{tot} = \tilde{S}_1 + \tilde{S}_2 = \underline{1010 - j\,15.6} \quad \text{VA} \quad (= 1010.1 \angle -0.88)$$

(b) $|\tilde{S}_s| = \frac{1}{2} \underset{\sqrt{2}V_{rms}}{V_m} \underset{\sqrt{2} I_{rms}}{I_m} \rightarrow I_{rms} = \frac{|S_s|}{V_{rms}} = \frac{|1010 - j15.6|}{120} = 8.42$ A rms

given

(c) $|\tilde{S}_1| = \frac{1}{2} V_{m1} I_{m1} = (120) \cdot I_{rms,1}$ $\qquad I_{1,rms} = \frac{1111.1}{\underset{V_{rms1}}{120}} = 9.26$ A (rms)

~~$V_m = 120$~~

$$1111.1 = V_{rms1} I_{rms1}$$

(d) $\tilde{S}_s = \frac{1}{2} \tilde{V} \tilde{I}^* = \frac{1}{2} \tilde{Z} \tilde{I} \tilde{I}^* = \frac{1}{2} |I|^2 \tilde{Z}$ $\qquad \tilde{V} = \tilde{I}\tilde{Z}$

$$\tilde{Z} = \frac{2 \tilde{S}_s}{|I|^2} = \frac{2(1010 - j15.6)}{(\sqrt{2} \cdot 8.42)^2} = \frac{1010 - j15.6}{(8.42)^2}$$

$I_m^2 = (\sqrt{2})^2 (8.42)^2$

$$= \underline{14.25 - j0.22}\ \Omega$$

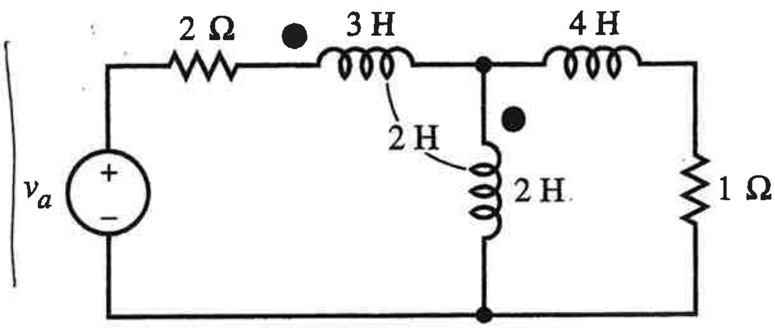
$$(= 14.25 \angle -0.88)$$

#6 Alternative

EE 2120 Final Exam Dec. 17, 2020

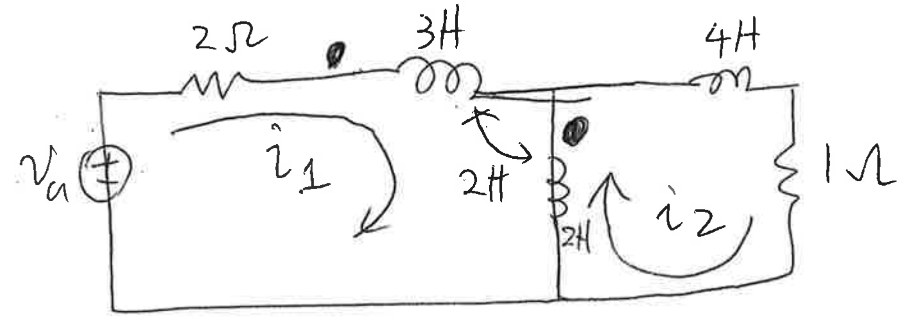
Name: ____________________

(6) Write two mesh current equations for the following circuit with mutual inductance M=2H as indicated in the following figure.



Ans:

$$\begin{bmatrix} 9\frac{d}{dt}+2 & -4\frac{d}{dt} \\ -4\frac{d}{dt} & 1+6\frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \end{bmatrix}$$



KVL mesh 1

$$-v_a + i_1 2 + 3\frac{di_1}{dt} + 2\frac{d}{dt}(i_1 - i_2)$$

$$+ \underbrace{2\frac{d}{dt}(i_1 - i_2)}_{\text{mutual inductance from 2H}} + \underbrace{2\frac{di_1}{dt}}_{\text{Mutal inductance for 3H}} = 0$$

KVL mesh 2

the same sign

$$4\frac{di_2}{dt} + i_2 + 2\frac{d}{dt}(i_2 - i_1) + \underbrace{2\frac{d(-i_1)}{dt}}_{\text{mutal inductam}} = 0$$

Name KEY

(7) The transfer function in s domain, H(s) of a circuit is given by

$$H(s) = \frac{I_o}{V_s} = \frac{(s+2)^2}{(s+3)(s^2+2s+10)}$$

a. If the input $v_s(t) = 5e^{-3t}\cos(2t + 30^0)$V, find the output $i_o(t)$. **(15 points)**

b. Identify the location of poles and zeros and sketch the pole-zero plot for the transfer function. **(10 points)**

$V_s(s) = 5\angle 30^\circ$ V

$s = -3 + j2$

(a) $I_o = H(s)\, V(s)$

$$= \frac{(-3+j2+2)^2}{(-3+j2+3)((-3+j2)^2 + 2(-3+j2) + 10)} \times 5\angle 30^\circ.$$

$= 1.038\angle -145.24^\circ$ A

$i_o(t) = 1.038\, e^{-3t} \cos(2t - 145.24^\circ)$ A.

(b) Location of zero: $S_{z_1} = -2$ (twice).

Location of Poles: $S_{p_1} = -3$

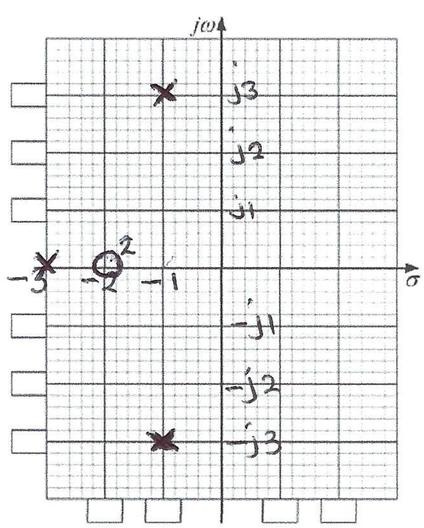
$S_{p_2} = -1 + j3$ and $S_{p_3} = -1 - j3$.

$i_o(t) =$ $1.038\, e^{-3t} \cos(2t - 145.24^\circ)$ A.

Location of poles: -2 (twice)

Location of zeros: -3, $-1 + j3$ and $-1 - j3$.

Pole-zero plot:



K=1

Problem Score

/25

Name KEY

(8) The transfer function in frequency domain of a circuit, $H(j\omega)$ is given by

$$H(j\omega) = \frac{V_{out}}{I_{in}} = \frac{(1 + j3\omega)}{(18 - 48\omega^2 + j20\omega)}$$

(a)

a. Develop the second order differential equation which models the circuit. **(10 points)**
b. If the input $i_{in}(t) = 50\sin(10t + 90^0)$A, find the output $v_{out}(t)$. **(15 points)**

$$H(jw) = \frac{V_{out}}{I_{in}} = \frac{(1 + j3w)}{(18 - 48w^2 + j20w)}$$

$$18 + 48(jw)^2 + 20(jw)V_{out} = (1 + 3(jw))I_{in}$$

$$jw \rightarrow D \rightarrow \frac{d}{dt}$$

$$\left(48\frac{d^2}{dt^2} + 20\frac{d}{dt} + 18\right)V_{out} = \left(1 + 3\frac{d}{dt}\right)i_{in}$$

(b) $i_{in}(t) = 50\sin(10t + 90^\circ)$ Amperes $= 50\cos 10t$ A.

$I_{in} = 50\angle 0^\circ$ A $\quad w = 10$ rad/s.

$$V_{out} = H(jw) \cdot I_{in}$$

$$= \frac{(1 + j3(10))}{(18 - 48(10)^2 + j(20)(10))} \times 50\angle 0^\circ$$

$$V_{out} = 0.314\angle -89.51^\circ V$$

$$v_{out} = 0.314\cos(10t - 89.51^\circ) V.$$

$$= 0.314\sin(10t - 0.5^\circ) V$$

Differential Equation = $\left(48\frac{d^2}{dt^2} + 20\frac{d}{dt} + 18\right)v_{out} = \left(1 + 3\frac{d}{dt}\right)i_{in}$

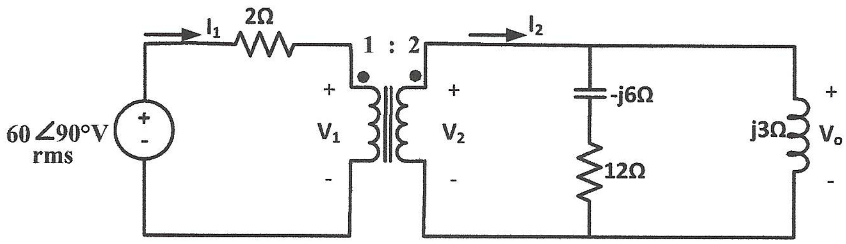
$v_{out}(t) =$ $0.314\cos(10t - 89.51^\circ)$ V or $0.314\sin(10t - 0.5^\circ)$ V

Problem Score

/25

Name KEY

(9) For the ideal transformer circuit below,



a. Find the currents $\mathbf{I_1}$ and $\mathbf{I_2}$. **(10 Points)**
b. Find the voltage $\mathbf{V_1}$, $\mathbf{V_2}$ and $\mathbf{V_o}$. **(10 Points)**
c. The complex power supplied by the source $\tilde{S}$. **(5 Points)**
(Note: Your answers should be in the **polar form**)

(a)

$$Z_L = j3 \| (12 - j6) = \frac{(j3)(12-j6)}{(12-j3)} = 3.25\angle 77.47\Omega \; (0.706 + j3.177\Omega)$$

$$Z_{in} = 2 + \frac{0.706 + j3.177}{2^2} = 2.176 + j0.794\Omega = 2.317\angle 20.04^\circ\Omega$$

$$I_1 = \frac{V_s}{Z_{in}} = \frac{60\angle 90^\circ}{2.317\angle 20.04} = 25.9\angle 69.96^\circ A.$$

$$I_2 = \frac{I_1}{2} = 12.95\angle 69.96^\circ A.$$

(b)

$$60\angle 90^\circ = 2I_1 + V_1$$

$$V_1 = (60\angle 90^\circ) - 2(25.9\angle 69.96^\circ)$$

$$= 21.06\angle 147.44^\circ V$$

$$V_2 = 2V_1 = 42.12\angle 147.44^\circ V$$

$$V_o = V_2 = 42.12\angle 147.44^\circ V$$

(c) $\tilde{S} = V_s I_1^* = (60\angle 90^\circ)(25.9\angle -69.96)$

$$= 1554\angle 20.04 VA.$$

I_1 = $25.9\angle 69.96^\circ A$.

I_2 = $12.95\angle 69.96^\circ A$.

V_1 = $21.06\angle 147.44^\circ V$

V_2 = $42.12\angle 147.44^\circ V$

V_o = $42.12\angle 147.44^\circ V$

$\tilde{S}$ = $1554\angle 20.04 VA$.

Problem Score
/25

KEY

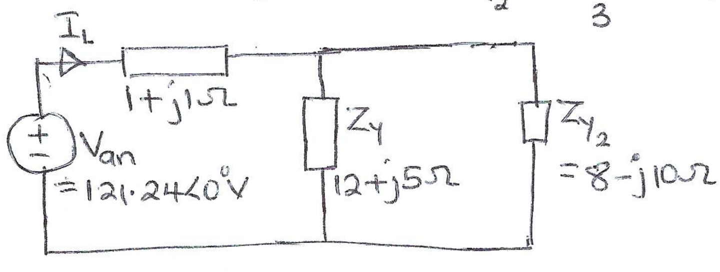
(10) A balanced three-phase system consists of a source with a line-to-line voltage of 210V connected to the parallel combination of a delta-load $Z_\Delta = 24 - j30\Omega$ and a wye-load $Z_Y = 12 + j5\Omega$ through a line impedance $Z_{line} = 1 + j1\Omega$.

a. Draw the per-phase equivalent circuit representation. Take phase A line-to-neutral voltage to be your angle reference. **(7 points)**

Converting the delta load into a wye load; $Z_{Y_2} = \frac{Z_\Delta}{3} = \frac{24-j30}{3} = 8-j10\Omega$

$V_{AN} = \frac{210}{\sqrt{3}}\angle 0^\circ V$

$= 121.24\angle 0^\circ V$



b. Determine the magnitude of the line current of the combined loads. **(8 points)**

$Z_p = Z_Y \| Z_{Y_2} = \frac{(12+j5)(8-j10)}{20-j5} = 8.076\angle -14.68^\circ\Omega \ (7.812 - j2.047\Omega)$

$Z_T = Z_p + Z_{line} = 8.812 - j1.047 = 8.874\angle -6.78^\circ\Omega$.

$I_L = \frac{V_{an}}{Z_T} = \frac{210}{(\sqrt{3} \times 8.874\angle -6.78^\circ)} = 13.66\angle 6.78^\circ A$

$|I_L| = 13.66A$

c. Determine the magnitude of the current flowing through the delta-connected load. **(5 points)**

$I_\Delta = \frac{12+j5}{(12+j5)+(8-j10)} \times 13.66\angle 6.78^\circ$ (By Current division)

$= 8.614\angle 43.44^\circ A$

$|I_\Delta| = 8.614A$

d. Determine the magnitude of the current flowing through the wye-connected load. **(5 points)**

$I_Y = \frac{8-j10}{(12+j5)+(8-j10)} \times 13.66\angle 6.78^\circ$ (By Current division

$= 8.486\angle -30.52^\circ A$.

$|I_Y| = 8.486A$

$|I_L|=$ 13.66A. $|I_\Delta| =$ 8.614A $|I_Y| =$ 8.486A.