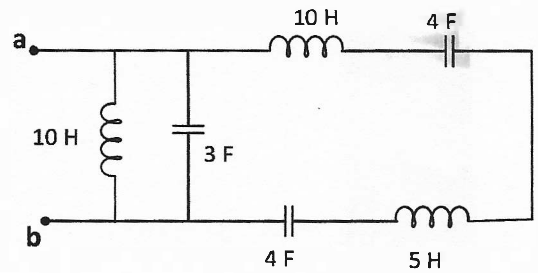
Name ________________________

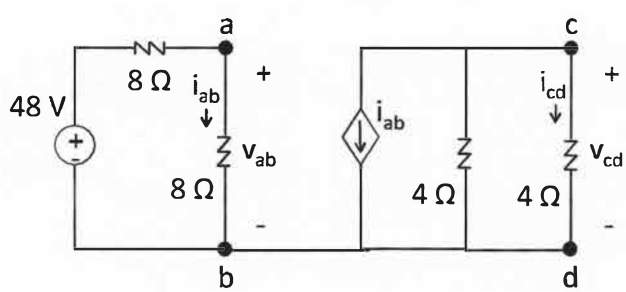
(1) **a)** Determine the equivalent capacitance (C_{ab}) and the equivalent inductance (L_{ab}) seen from the terminals a – b. (9pts)



$C_{ab} = 3F \| [4F - 4F] = 3 + 2 = \boxed{5F}$

$L_{ab} = 10H \| [10H - 5H] = \frac{10(15)}{25} = \boxed{6H}$

b) In the circuit below, calculate the unknown currents and voltages: i_{ab}, v_{ab}, i_{cd}, and v_{cd}. (16pts)



Voltage divider: $V_{ab} = \frac{8}{8+8} \cdot 48 = \boxed{24V}$

Ohm's law $i_{ab} = \frac{V_{ab}}{8} = 24/8 = \boxed{3A}$

Current divider $i_{cd} = \frac{4}{4+4}(-i_{ab}) = \boxed{-1.5A}$

Ohm's law $V_{cd} = 4\, i_{cd} = 4(-1.5) = \boxed{-6V}$

C_{ab}= 5F ______ L_{ab}= 6H ______

i_{ab} = 3A ______ v_{ab} = 24V ______ i_{cd} = -1.5A ______ v_{cd} = -6V ______

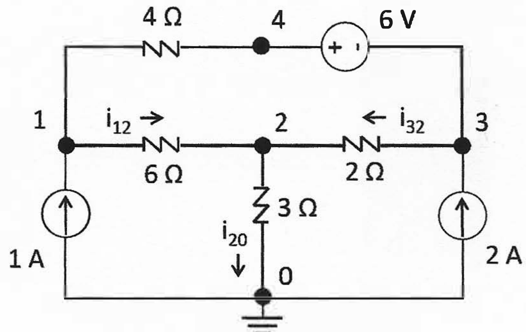
Problem Score
/25

Name ____________________

(2) For the circuit below,

a) Write a set of node voltage equations for the following circuit in matrix form. (13 pts)

b) Given that $v_1 = 17$ V and $v_2 = 9$ V, calculate i_{12}, i_{20}, and i_{32}. (12 pts)



V_1, V_2, V_3, V_4

V_{src}: $V_4 - V_3 = 6$

$\boxed{-V_3 + V_4 = 6} \longrightarrow$ (i)

KCL@1: $\dfrac{V_1 - V_4}{4} + \dfrac{V_1 - V_2}{6} = 1 \quad \times 12$

$\boxed{5V_1 - 2V_2 - 3V_4 = 12} \longrightarrow$ (ii)

KCL@2: $\dfrac{V_2 - V_1}{6} + \dfrac{V_2}{3} + \dfrac{V_2 - V_3}{2} = 0 \quad \times 6$

$\boxed{-V_1 + 6V_2 - 3V_3 = 0} \longrightarrow$ (iii)

KCL@0: $1 - \dfrac{V_2}{3} - 2 = 0 \Rightarrow \boxed{V_2 = 9} \longrightarrow$ (iv)

OR KCL@SN(3-4): $\dfrac{V_3 - V_2}{2} + \dfrac{V_4 - V_1}{4} = 2 \quad \times 4$

$\boxed{-V_1 - 2V_2 + 2V_3 + V_4 = 8} \longrightarrow$ (iv)

$$\begin{bmatrix} 0 & 0 & -1 & 1 \\ 5 & -2 & 0 & -3 \\ -1 & 6 & -3 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 0 \\ 9 \end{bmatrix}$$

OR $-1 \quad -2 \quad 2 \quad 1 \qquad 8$

$i_{12} = \dfrac{V_1 - V_2}{6} = \dfrac{17-9}{6} = \dfrac{4}{3}$ A

$i_{20} = \dfrac{V_2}{3} = \dfrac{9}{3} = 3$ A

KCL@2:

$i_{32} + i_{12} - i_{20} = 0$

$\therefore i_{32} = i_{20} - i_{12} = 5/3$ A

i_{12} = 4/3 A $\qquad i_{20}$ = 3 A $\qquad i_{32}$ = 5/3 A

Problem Score
/25

Name ______________

(3) For the circuit below,

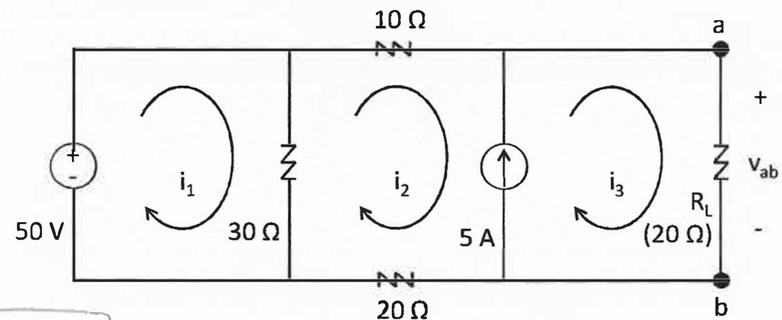
a) Write a set of mesh current equations for the following circuit in matrix form. (13 pts)

b) Given that i_2 = -1A and i_3 = 4 A, calculate the following:

i. Voltage v_{ab} (4pts)

ii. Power **absorbed** by 5 A current source [note: *no credit with wrong power sign convention*]. (4pts)

iii. Power **absorbed** by the resistor R_L (= 20 ohm) [note: *no credit with wrong power sign convention*]. (4pts)



$\boxed{-i_2 + i_3 = 5} \longrightarrow (1)$

KVL @ i_1 : $-50 + 30i_1 - 30i_2 = 0 \Rightarrow \boxed{30i_1 - 30i_2 = 50} \longrightarrow (ii)$

KVL @ SM(2-3) : $30i_2 - 30i_1 + 10i_2 + 20i_3 + 20i_2 = 0$

$\therefore \boxed{-30i_1 + 60i_2 + 20i_3 = 0} \longrightarrow (iii)$

$V_{ab} = i_3 \cdot 20 = 20(4) = \boxed{80V}$

$P_{5A} = -5 V_{ab} = -5(80) = \boxed{-400W}$ $\quad P_{RL} = \frac{V_{ab}^2}{R_L}$ or $V_{ab} i_3 = 4(80) = \boxed{320W}$

$$\begin{bmatrix} 0 & -1 & 1 \\ 30 & -30 & 0 \\ -30 & 60 & 20 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \\ 0 \end{bmatrix}$$

v_{ab} = 80V   power (5A source) = -400W   power (R_L) = 320 W

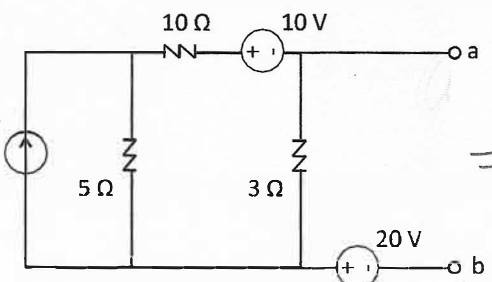
Problem Score
/25

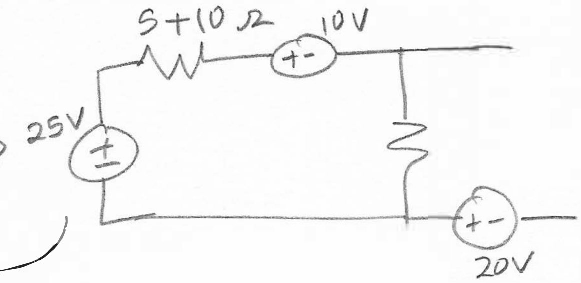
Name ______

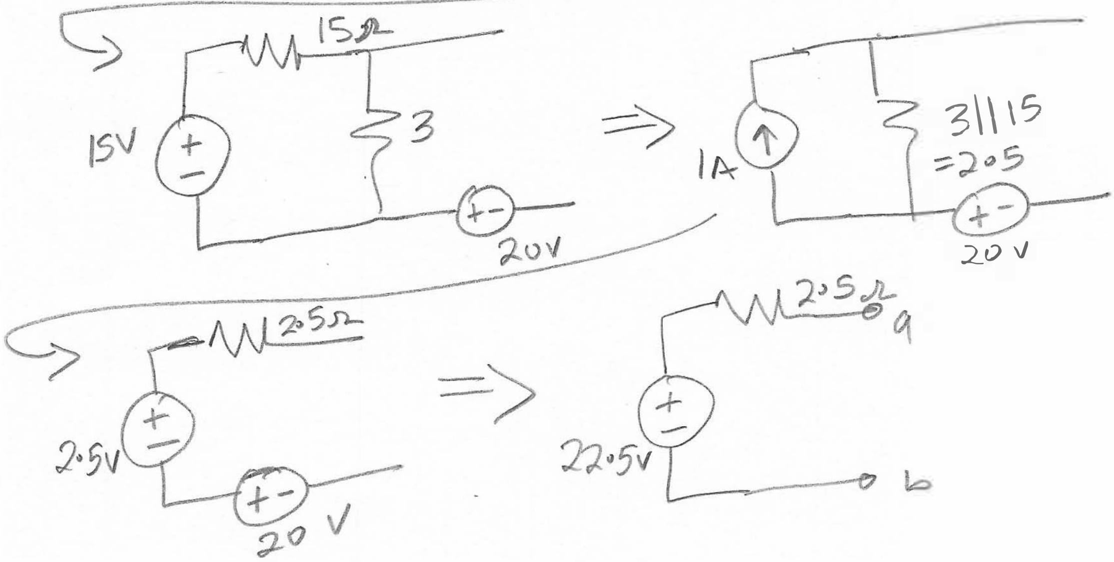
(4) For the network below,

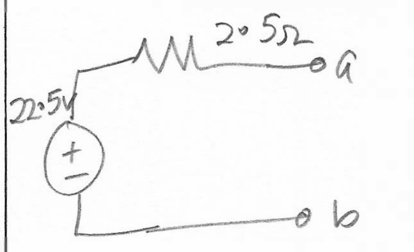
(a) Use a series of source transformations to determine and sketch the equivalent Thevenin circuit with respect to terminals a and b consisting of a voltage source and series resistance. (20 pts)

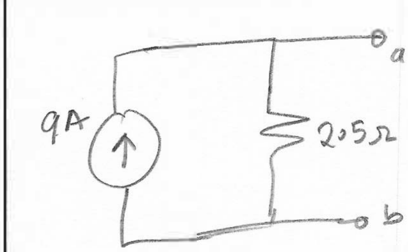
(b) Sketch the equivalent Norton circuit based on your answer obtained in part (a). (5 pts)








Thevenin Equivalent Circuit


Norton Equivalent Circuit

Problem Score

/25

Name T

(5) For the circuit below,

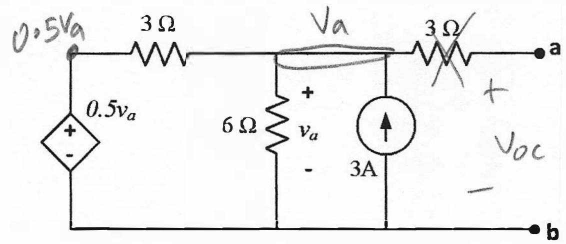
a. Find and sketch the Thevenin equivalent circuit with respect to the terminals a and b. (15 pts)

b. Find the load resistance, R_L, which should be attached between the end terminals a and b so that maximum power is delivered to the load. (3 pts)

c. Calculate the value of the maximum power delivered to the load. (7pts)

$R_L = R_{th} = 6\,\Omega$

$P_{max} = \dfrac{V_{th}^2}{4R_{th}} = \dfrac{9^2}{4(6)} = \boxed{3.375\,W}$



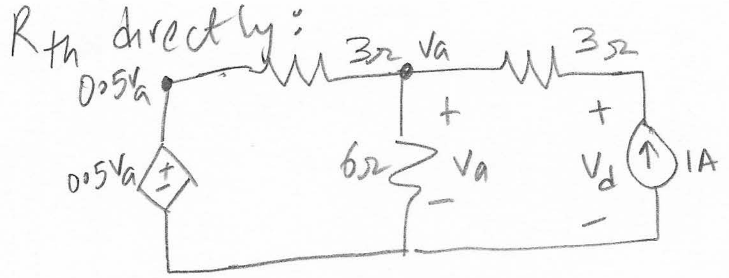
V_{th} directly

$V_{oc} = V_a$

KCL @ V_a: $\dfrac{V_a - 0.5V_a}{3} + \dfrac{V_a}{6} - 3 = 0 \Rightarrow 2V_a = 18 \Rightarrow V_a = 9V$

$\therefore \boxed{V_{oc} = 9V}$

R_{th} directly:



$R_{th} = \dfrac{V_d}{1}$

KVL: $-V_a - 3 + V_d = 0$

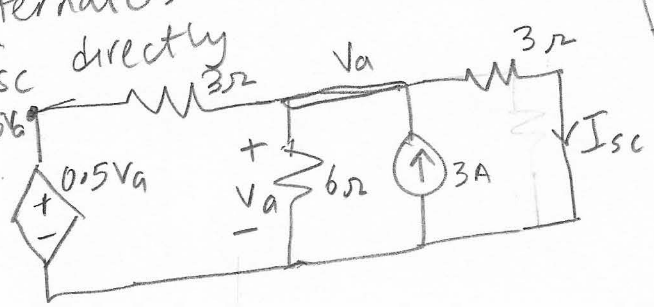
$V_d = 3 + V_a \rightarrow (1)$

KCL @ V_a: $\dfrac{V_a}{6} + \dfrac{V_a - 0.5V_a}{3} = 1 \Rightarrow 2V_a = 6 \Rightarrow V_a = 3V \quad \therefore V_d = 6V$

$\therefore \boxed{R_{th} = 6\,\Omega}$

Alternate:

I_{sc} directly



$\boxed{I_{sc} = V_a/3}$

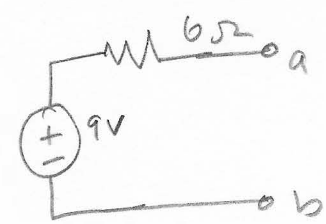
KCL @ V_a:

$\dfrac{V_a}{6} + \dfrac{V_a}{3} + \dfrac{V_a - 0.5V_a}{3} = 3 \Rightarrow 4V_a = 18$

$\boxed{V_a = 18/4}$

$\therefore I_{sc} = 18/12 = \boxed{3/2\,A}$

Thevenin Equivalent Circuit



$\therefore R_{th} = \dfrac{V_{oc}}{I_{sc}} = \boxed{6\,\Omega}$

same

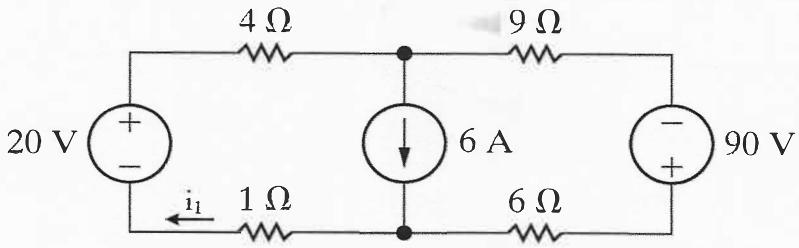
$R_L =$ 6Ω

$P_{max} =$ 3.375W

Problem Score / 25

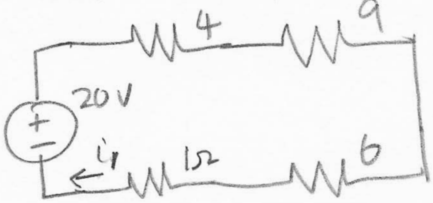
Name ______________________________

(6) Use superposition to calculate the current, i_1, in the following circuit. Sketch the circuits for each step of the superposition. (25 pts.)



$$i_1 = i_1|_{20V} + i_1|_{6A} + i_1|_{90V}$$

$i_1|_{20V}$:

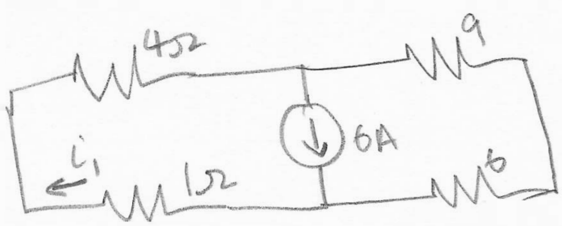


$\Rightarrow$

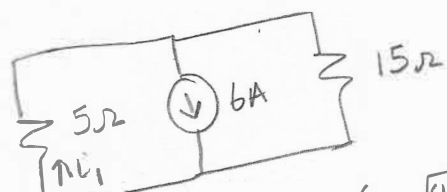
Ohm's law

$$i_1 = \frac{20}{20} = \boxed{1A}$$

$i_1|_{6A}$:

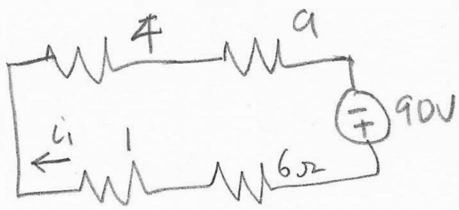


$\Rightarrow$

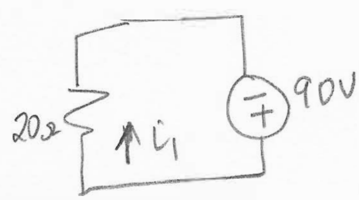


current divider: $i_1 = \frac{15}{15+5} \cdot 6 = \boxed{4.5A}$

$i_1|_{90V}$:



$\Rightarrow$



Ohm's law

$$i_1 = \frac{90}{20} = \boxed{4.5A}$$

$$\therefore i_1 = 1 + 4.5 + 4.5$$
$$= \boxed{10A}$$

i_1= 10A

Problem Score /25

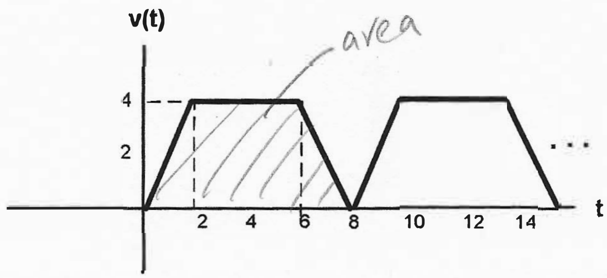
Name ________________________________

(7) Given the following voltage signal;

a. Write an expression for v(t) utilizing the step unit and ramp functions over the interval 0 < t < T where T is the period.

b. Calculate the average voltage V_{avg} over the period (10 pts.)

c. Calculate the average power dissipation P_{av} when v(t) is applied across a resistor of 4 ohms. (8 pts.)



$T = 8$

(a) $v(t) = 2r(t) - 2r(t-2) + 2r(t-6)$

(b) $V_{avg} = \frac{1}{T}\int_T v(t)\,dt = \frac{1}{8}\left[\int_0^2 2t\,dt + \int_2^6 4\,dt + \int_6^8 (16-2t)\,dt\right]$

$= \frac{1}{8}(8+16) = \boxed{3V}$ $\qquad \frac{\text{area under graph}}{T}$ also valid

(c) $P_{av} = \frac{V_{rms}^2}{4}$; $V_{rms} = \sqrt{\frac{1}{T}\int v^2\,dt} = \sqrt{\frac{1}{8}\left(\int_0^2 4t^2\,dt + \int_2^6 16\,dt + \int_6^8 (16-2t)^2\,dt\right)}$

$\therefore V_{rms}^2 = \frac{1}{8}\left(10.67 + 64 + 10.67\right) = 10.67$

$\therefore P_{av} = \frac{10.67}{4} = \boxed{2.67\,W}$

v(t) = $2r(t) - 2r(t-2) - 2r(t-6)$

V_{avg} = 3V

P_{av} = 2.67W

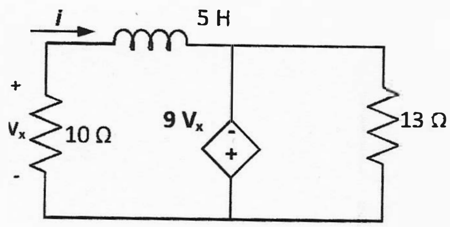
Problem Score

/25

Name ______________________

(8) For the source-free circuit below, given that $i(0^+) = 80\ A$, determine the following:

a. The differential equation in terms of i(t). (15 pts.)
b. The time constant τ (3 pts.)
c. The current through the inductor i(t) for t>0. (7 pts.)



KVL @ left loop

$$-V_x + 5\frac{di}{dt} - 9V_x = 0$$

$$\frac{di}{dt} - 2V_x = 0 \quad —(1)$$

from ohm's law

$$V_x = -10i$$

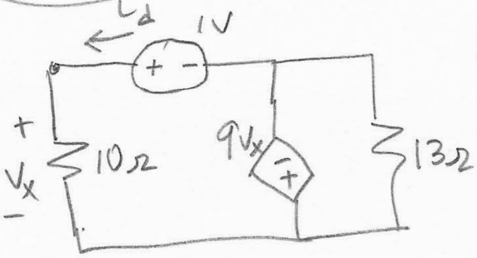
$$\therefore \boxed{\frac{di}{dt} + 20i = 0}$$

b) $\tau = 1/20\ s$

c) $i(t) = Ae^{-t/\tau} = Ae^{-20t}$; $A = i(0^+) = 80$

$$\therefore \boxed{i(t) = 80e^{-t/\tau}\ A}$$

Alternate simplify ckt : using R_{th}



$R_{th} = 1/i_d$

KVL : left loop $-V_x + 1 - 9V_x = 0$

$V_x = 1/10$

ohm's law : $\frac{V_x}{10} = i_d \Rightarrow i_d = 1/100$

$\therefore R_{th} = 1/i_d = \boxed{100\Omega}$

∴ simplified ckt

$\Rightarrow \tau = L/R = 5/100$

$= \boxed{1/20\ s}$ same

differential equation: $\frac{di}{dt} + 20i = 0$

time constant τ = $1/20$ s

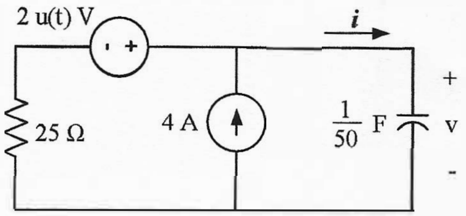
i(t) = $80\ e^{-t/\tau}$ A

Problem Score
/25

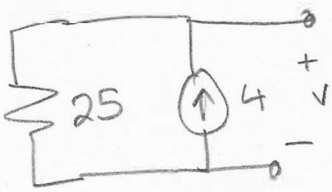
Name ______________________

(9) For the circuit below,

a. Determine the initial condition $v(0^+)$. (5 pts.)
b. Write the differential equation for the circuit in terms of $v(t)$ for $t>0$. (10pts.)
c. Determine the complete solution, $v(t)$ for $t>0$. (10 pts.)



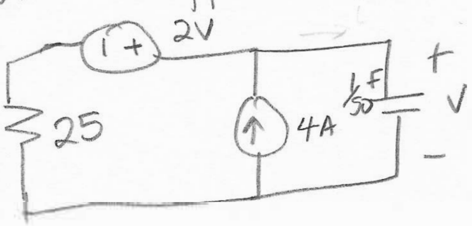
a) $v(0^+) = v(0^-)$ go to ckt $t<0$ i.e. $t=0^-$



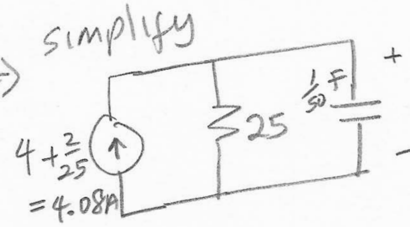
$= 100V$ (Ohm's law)

$\therefore \boxed{V(0^+) = 100V}$

b) $t>0$ diff eqn?



$\Rightarrow$ simplify



KCL @ top node

$\frac{1}{50}\frac{dv}{dt} + \frac{V}{25} = 4.08$

$\boxed{\frac{dv}{dt} + 2V = 204}$

c) $v(t)$? $v(t) = V_n + V_p$

$V_n = Ae^{-t/\tau} = Ae^{-2t}$; $V_p = K \therefore V_p' = K$

plug into diff eqn

$2K = 204 \Rightarrow K = 102$

$\therefore \boxed{V_p = 102V}$

$\therefore V(t) = Ae^{-2t} + 102$

use I.C to find A: $v(0) = A + 102 = 100$

$\therefore A = -2 \quad \therefore \boxed{V(t) = -2e^{-2t} + 102\ V}$

$v(0^+) =$ 100 V

differential equation: $v' + 2v = 204$

$v(t) =$ $-2e^{-2t} + 102$ V

Problem Score
/25

Name ______ ____________________

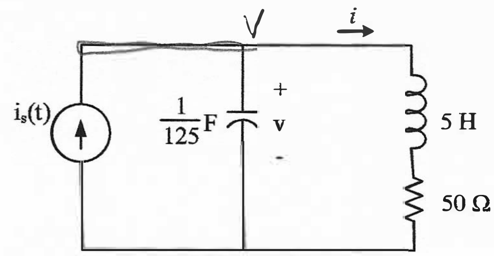
(10) For the following RLC circuit, given that $i_s = 12e^{-3t}$ A t>0. Do the following:

a. Write a differential equation, in terms of i, to find the current through the inductor t>0. (13pts)

b. Find the form of the natural response $i_n(t)$ t>0. (5 pts)

c. Solve for the particular solution $i_p(t)$ t>0. (5pts)

d. Solve for the general form of complete solution $i(t)$ t>0 in terms of the two undetermined coefficients only. (2 pts)



a) KCL @ V:

$$\frac{1}{125}\frac{dv}{dt} + i = i_s \quad \text{—(I)}$$

KVL @ R Loop

$$-V + 5\frac{di}{dt} + 50i = 0$$

$$\therefore \boxed{V = 5\frac{di}{dt} + 50i} \quad \text{—(II)}$$

$\therefore$ plug (II) into (I)

$$\frac{1}{125}\left(5i'' + 50i'\right) + i = i_s \quad \times 25$$

$$i'' + 10i' + 25i = 25 i_s$$

$$\boxed{i'' + 10i' + 25i = 300e^{-3t}}$$

b) $i_n(t)$? Characteristic eqn: $s^2 + 10s + 25 = 0$

$s_{1,2} = -5$ repeated roots

$$\boxed{\therefore i_n(t) = A_1 e^{-5t} + A_2 t e^{-5t} \text{ A}}$$

c) $i_p(t)$? $i_p = Ke^{-3t} \Rightarrow i_p' = -3Ke^{-3t}$; $i_p'' = 9Ke^{-3t}$

plug into diff eqn: $9Ke^{-3t} + 10(-3K)e^{-3t} + 25Ke^{-3t} = 300e^{-3t}$

$\therefore 4K = 300 \Rightarrow K = 75 \quad \therefore \boxed{i_p = 75e^{-3t} \text{ A}}$

d) $i(t) = i_n(t) + i_p(t) = \boxed{A_1 e^{-5t} + A_2 t e^{-5t} + 75e^{-3t} \text{ A}}$

differential equation: $i'' + 10i' + 25i = 300e^{-3t}$

$i_n(t) =$ $A_1 e^{-5t} + A_2 t e^{-5t}$ A $\quad i_p(t) =$ $75e^{-3t}$ A

$i(t) =$ $A_1 e^{-5t} + A_2 t e^{-5t} + 75e^{-3t}$ A

Problem Score
/25