EE3501, Experiment 2
EE3541, Experiment 4
Single-Phase Transformers and AC Excitation

Jonathan Kimball, August 31, 2020

Abstract

In this lab, students will explore the characteristics of a (single-phase) transformer, including the nonlinear nature of ac excitation and linear models. The result will be a model of a physical transformer that may be used in future circuit analysis.

1   Introduction

Transformers are foundational elements in all power distribution systems. A transformer couples two or more coils to the same magnetic flux. As long as the flux is changing (ac), power may be transferred from one coil to another. Unfortunately, changing flux in a magnetic material gives rise to losses, as well as changes in stored energy.

Figure 1 comprises a standard transformer symbol, which may be used to indicate a real or ideal transformer. Figure 2 comprises the Steinmetz (or T-equivalent) model, which incorporates non-ideal characteristics of a real transformer. The relevant variables are:

$\hat{V}_1$: Primary side voltage [V]

$\hat{I}_1$: Primary side current [A]

$\hat{V}_2$: Secondary side voltage [V]

$\hat{I}_2$: Secondary side current [A]

$\hat{V}_2'$: Secondary side voltage, referred to primary side [V]; $\hat{V}_2' = a\hat{V}_2$

$\hat{I}_2'$: Secondary side current, referred to primary side [A]; $\hat{I}_2' = \frac{1}{a}\hat{I}_2$

$\hat{I}_\phi$: Magnetizing current [A]; sometimes $\hat{I}_{\phi 1}$ to indicate primary side

The model parameters are:

R_1: Primary coil resistance [Ω]

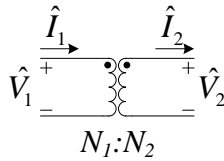$X_{\ell 1}$: Primary coil leakage reactance [Ω]
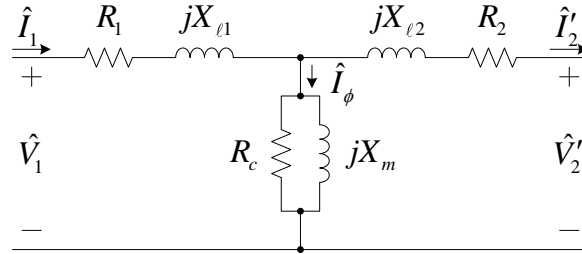
Figure 1: Transformer symbol.

Figure 2: Steinmetz (T-equivalent) model of a transformer.

R_c: Equivalent core loss resistance [Ω]; sometimes R_{c1} to indicate primary side

X_m: Equivalent core magnetizing reactance [Ω]; sometimes X_{m1} to indicate primary side

R_2, R_2': Secondary coil resistance [Ω], actual or referred; $R_2' = a^2 R_2$

$X_{\ell 2}$, $X_{\ell 2}'$: Secondary coil reactance [Ω], actual or referred; $X_{\ell 2}' = a^2 X_{\ell 2}$

N_1, N_2: Turns on primary and secondary; must be positive integers

$a = {N_1}/{N_2}$: Transformer turns ratio; must be rational

The turns ratio is generally given as a voltage ratio. For example, a transformer might be rated 240:120 V, meaning $a = 2$.

The Steinmetz model incorporates linearized forms of the nonlinear loss and energy storage characteristics of the device. R_1 and R_2 are simply ohmic terms and inherently linear, though temperature dependent; $X_{\ell 1}$ and $X_{\ell 2}$ are leakage terms and essentially linear; but R_c and X_m represent highly nonlinear physical processes and are valid near the conditions where test data is obtained.

1.1   Measuring Magnetizing Apparent Power and Core Losses

All practical magnetic materials are nonlinear, with hysteresis and saturation, and have finite permeability. These material characteristics result in magnetizing current I_ϕ that is non-sinusoidal and partly in-phase with the applied voltage. To measure this effect, an open-circuit test is performed in which voltage is applied to one terminal of the transformer and the other terminal is left open. Then the Steinmetz model degenerates as shown in Figure 3.

As noted, this model is only valid over a limited operating range. In *Fitzgerald & Kingsley's Electric Machinery*, 7ed., by Stephen D. Umans, Figs. 1.12 and 1.14 show consumed apparent and active power (S_a and P_c) for a particular steel commonly used for transformers, measured over a wide range of conditions, given in VA/kg and W/kg vs. peak flux density $B_{\max}$ in T. When characterizing a particular device, such as the transformers in the Missouri S&T laboratory, we may use the fact that mass is constant and, from Faraday's law,
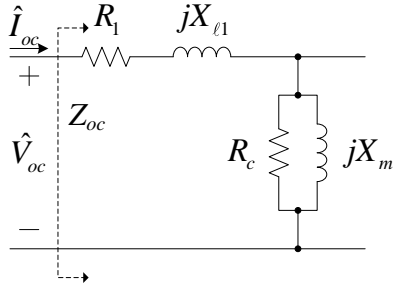
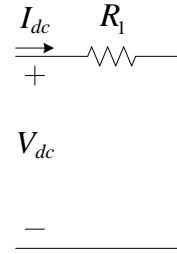$$V_{rms} = \sqrt{2}\pi f B_{\max} N A_c \tag{1}$$

Figure 3: Open-circuit test.

Figure 4: DC test.

(where f is the frequency of applied voltage, N is the number of turns, and A_c is the cross-sectional area of the core). Then we can plot power in VA or W vs. rms voltage and obtain plots of the same shape, scaled by the various geometrical parameters of the transformer. We may also probe i_1 and v_1 with an oscilloscope to witness the nonlinear characteristics.

1.2   Measuring Winding Resistance

The coil resistances R_1 and R_2 can be found by applying a dc voltage to the respective windings. At dc, the inductances all become short circuits, so the equivalent circuit is as shown in Figure 4. Then

$$R_1 = \frac{V_{dc}}{I_{dc}} \tag{2}$$

and similarly for R_2.

1.3   Measuring Leakage Reactances

If the secondary of the transformer is short-circuited, the magnetizing current $\hat{I}_\phi$ is negligible and the equivalent circuit is as shown in Figure 5. Using a power meter to find V_{sc}, I_{sc}, and P_{sc}, we may easily find the parameters:

$$S_{sc} = V_{sc}I_{sc} \tag{3}$$

$$Z_{sc} = \frac{V_{sc}}{I_{sc}} \tag{4}$$

$$\underline{/\bar{Z}_{sc}} = \arccos(\frac{P_{sc}}{S_{sc}}) \tag{5}$$

The two leakage reactances are connected in series, so

$$X_{\ell 1} + X'_{\ell 2} = \operatorname{Im} \bar{Z}_{sc} \tag{6}$$

A reasonable approximation then is

$$X_{\ell 1} = X'_{\ell 2} = \frac{1}{2} \operatorname{Im} \bar{Z}_{sc} \tag{7}$$

Naturally, the short-circuit test results should also give a real part of $\bar{Z}_{sc}$ consistent with the dc tests.
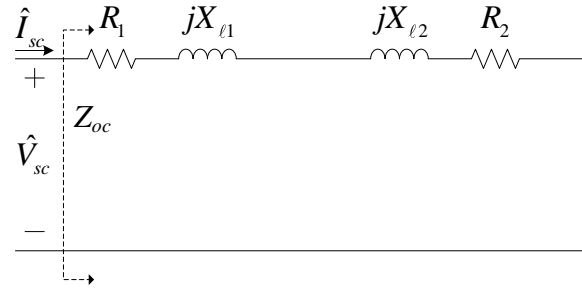
Figure 5: Short-circuit test.

1.4 Finding Magnetizing Reactance and Core Loss Resistance

From the open-circuit test above (Figure 3), we may find V_{oc}, I_{oc}, and P_{oc} Over a wide operating range, the relationships are all nonlinear. At a particular operating point, though, the characteristics may be approximated as linear. As for the short-circuit test,

$$S_{oc} = V_{oc}I_{oc} \tag{8}$$

$$Z_{oc} = \frac{V_{oc}}{I_{oc}} \tag{9}$$

$$\underline{/\bar{Z}_{oc}} = \arccos(\frac{P_{oc}}{S_{oc}}) \tag{10}$$

Given the short-circuit test results as well, the core admittance is

$$Y_\phi = \frac{1}{Z_{oc} - (R_1 + jX_{\ell 1}} \tag{11}$$

which may be split into real and imaginary parts to find the core parameters,

$$X_m = -\frac{1}{\operatorname{Im} Y_\phi} \tag{12}$$

$$R_c = \frac{1}{\operatorname{Re} Y_\phi} \tag{13}$$

1.5 Cantilever Approximation

Often, instead of using the Steinmetz T equivalent circuit, we use instead the approximate or cantilever equivalent circuit shown in Figure 6. Here, coil parameters have simply been added in series:

$$R_{eq} = R_1 + R_2' \tag{14}$$
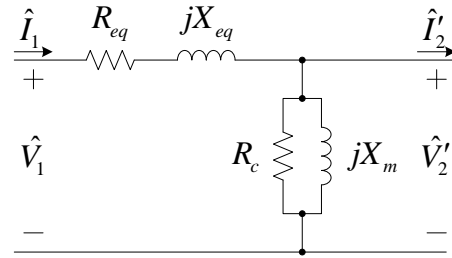$$X_{eq} = X_{\ell 1} + X_{\ell 2}' \tag{15}$$

Figure 6: Cantilever (approximate) equivalent circuit.

and may be placed on either side of the $(R_c \parallel jX_m)$ branch. We neglect $R_{eq} + jX_{eq}$ when evaluating the open-circuit tests. Then

$$R_{eq} = \frac{P_{sc}}{I_{sc}^2} \tag{16}$$

$$X_{eq} = \frac{Q_{sc}}{I_{sc}^2} \tag{17}$$

$$R_c = \frac{V_{oc}^2}{P_{oc}} \tag{18}$$

$$X_m = \frac{V_{oc}^2}{Q_{oc}} \tag{19}$$

where we have used $Q = \sqrt{S^2 - P^2}$. These results should be close to the results from the previous subsections.

2 Laboratory Software

In this experiment, you will use the Yokogawa to measure ac power, Fluke #1 to measure dc voltage and current, and the oscilloscope to view the shape of the ac voltage and current.

The Yokogawa should be in 1P3W mode throughout. Use channel 1.

Fluke #1 should be in dc mode, measuring both voltage and current.

Connect the current probe to one of the AM503B amplifiers, and its output to channel 1 of the oscilloscope. In LabView, ensure that this channel is set to be a current, and that the current scale matches the AM503B setting. Connect one of the P5205 differential probes to channel 2 of the oscilloscope. In LabView, ensure that this channel is set to be a voltage.

3 Laboratory Experiment

3.1 AC Excitation

Connect as in Figure 7. Notice that we are using the line-to-line voltage out of the variac, phase A to B, as a single-phase source.

The current probe is FRAGILE. Carefully place it on the appropriate wire pointing in the correct direction. Ensure that the probe is fully closed.
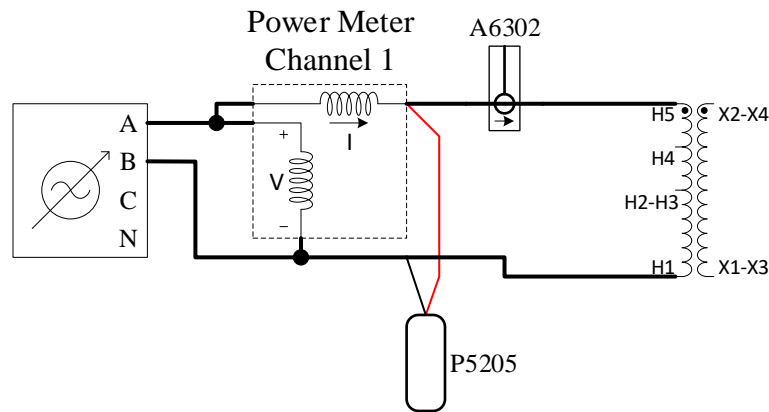
Figure 7: Open-circuit and ac excitation test.

With the variac reduced as close to zero as possible, switch it on. Record this data (both Yokogawa and scope). Increase the voltage in increments of about 5 V, recording the Yokogawa data throughout. At a few different points where the current waveform is qualitatively different, also record the scope waveform. Continue until you reach 100 % on the variac. Then switch off the variac and reduce its voltage to zero.

3.2   Open-Circuit Test

The first test needed to create a model of the transformer is the last data point from the AC excitation test. The data gathered at 100 % voltage is the open-circuit test point.

3.3   Short-Circuit Test

Set the load box as shown in Figure 8a. Change the primary connections as shown in Figure 9, and add a shorting wire as shown. Notice that the oscilloscope and probes are not used for this test.

Turn on the load box fans. With the variac at zero, switch it on. Slowly increase the voltage until the primary current matches the transformer rating of 4.17 A. Record the Yokogawa data, turn off the variac and reduce its voltage to zero.

3.4   DC Test

For the dc test, the load box remains in the same configuration, as shown in Figure 8a. However, the dc source and Fluke DMM are used instead, as shown in Figure 10.

There are two dc tests. In both cases, you will turn on the variable dc, slowly increase its voltage, record the Fluke data, then switch off the variable dc and reduce its output to zero.

- Connect to the H1 and H5 terminals of the transformer (primary), exactly as shown in Figure 10. Turn on and increase the variable dc until the current reads 4.17 A.

(a) R load   (b) RL load   (c) RC load
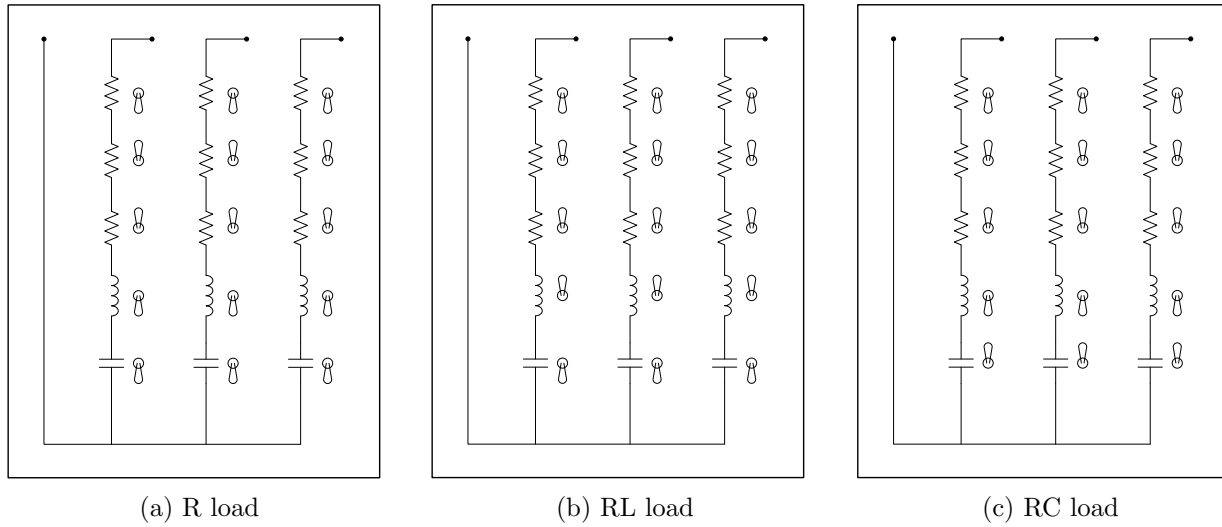
Figure 8: Load box settings. For short-circuit and dc tests, use the resistive load setting.
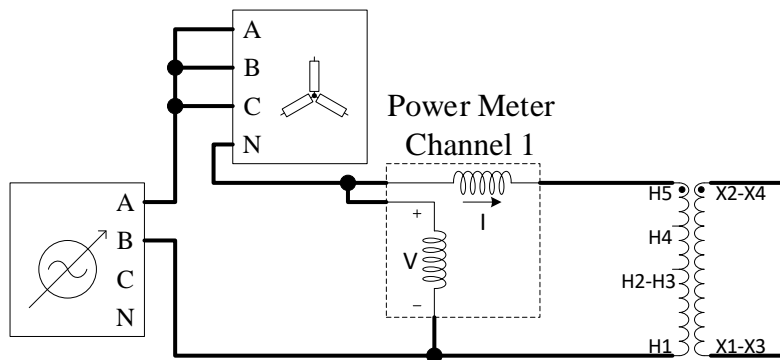

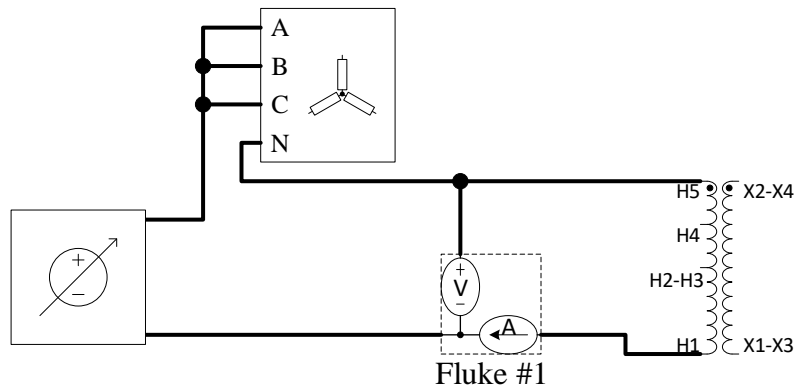Figure 9: Short-circuit test connections. Be sure load box is in "R" setting.

Figure 10: DC test connections. Be sure load box is in "R" setting.
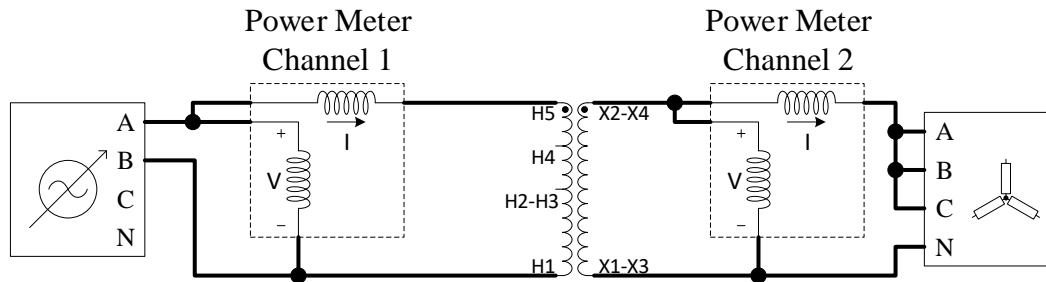
Figure 11: Load test connections.

- Connect to the X1-X3 and X2-X4 terminals of the transformer (secondary). Turn on and increase the variable dc until the current reads 8.33 A.

Remember to record the Fluke readings each time.

3.5   Load Test

Reconnect the system as shown in Figure 11. Perform three tests. For each test,

1. Turn on the variac

2. Increase to full voltage

3. Record the Yokogawa data for primary and secondary

4. Turn off the variac and reduce its output to zero

The load box settings for the three tests are:

- Resistive: as in Figure 8a

- R-L: as in Figure 8b

- R-C: as in Figure 8c

After this experiment is complete, you may turn off the load box fans.

4   Calculations and Question

1. From the first set of open-circuit tests, plot active power vs. applied voltage and apparent power vs. applied voltage. Attempt to replicate Figures 1.12 and 1.14 from the EE 3500 book, *Fitzgerald & Kingsley's Electric Machinery 7th Edition* (although with a different scale). Be sure to use a semilog grid (log scale on the x axis, linear scale on the y axis). Comment on any differences between your data and the curves in the book. For convenience, the figures are provided below as well.

2. In the book figures, especially Fig. 1.12, saturation is apparent–a substantial increase in power consumption for a small change in flux density. Most likely, your data will not show this same effect, at least not to the same degree. Why? Given the equipment available in the lab, how might you explore the saturated region?

3. From the dc, short-circuit, and open-circuit tests, calculate the Steinmetz model parameters (Figure 2).

4. Calculate the referred values of load impedance for the three load tests. Compute the expected secondary voltage and both primary and secondary currents based on the applied primary voltage for all three tests. Compare to the measured values, with percent errors.
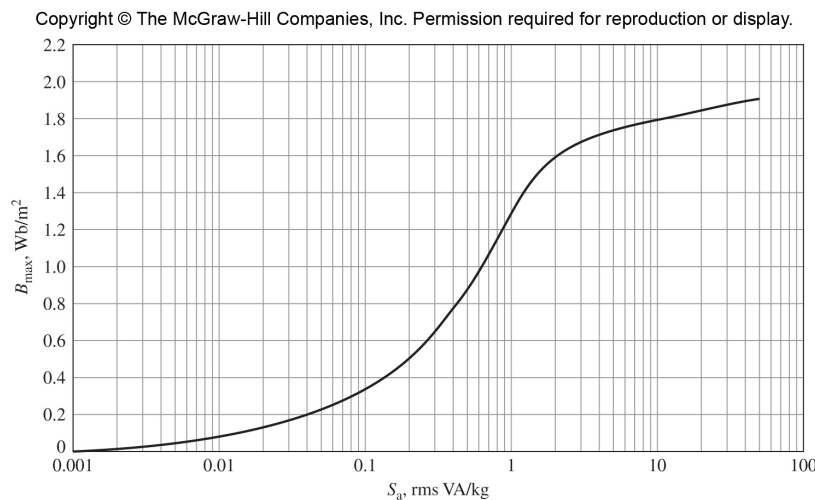
(a) Figure 1.12 from F&K.
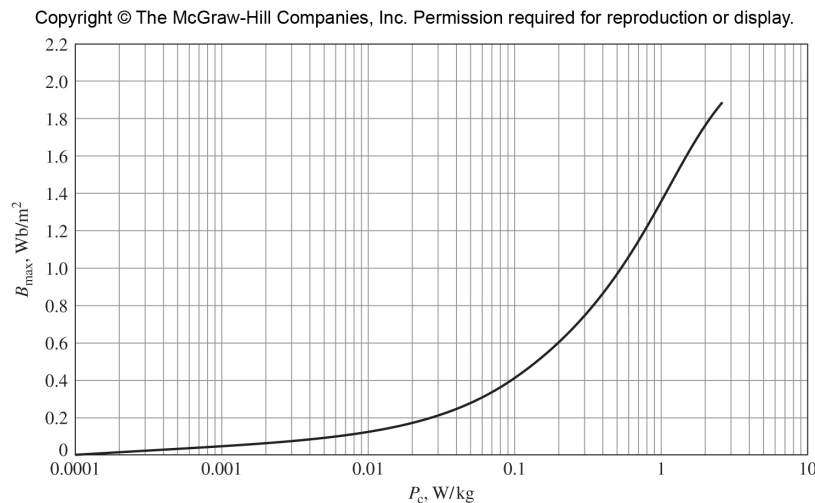
(b) Figure 1.14 from F&K.

Figure 12: Figures from *Fitzgerald & Kingsley's Electric Machinery 7th Edition*, provided for reference only.