

EE3501, Experiment 1

EE3541, Experiment 3

Power Measurements

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Abstract

In this lab, students will become familiar with single- and three-phase circuits, particularly in the context of equipment at Missouri S&T. In addition, students will learn how to perform power measurements for single-phase circuits (one wattmeter) and for three-phase circuits (two-wattmeter and three-wattmeter methods).

1 Introduction

In a single-phase circuit, the instantaneous voltage and current are

$$v_a(t) = \sqrt{2}V_{rms} \cos(\omega t) \quad (1)$$

$$i_a(t) = \sqrt{2}I_{rms} \cos(\omega t - \theta) \quad (2)$$

where V_{rms} and I_{rms} are the root mean square values of voltage and current, respectively; θ is the angle of the load impedance; and the radian frequency ω is related to the frequency in Hertz f by $\omega = 2\pi f$. Commonly, these time-domain waveforms are represented by complex vectors known as *phasors*,

$$\hat{V}_a = V_{rms} \angle 0 \quad (3)$$

$$\hat{I}_a = I_{rms} \angle -\theta \quad (4)$$

This is a simpler notation and enables us to perform some useful manipulations.

The instantaneous power is the product of instantaneous voltage and current,

$$p(t) = 2V_{rms}I_{rms} \cos(\omega t) \cos(\omega t - \theta) \quad (5)$$

This may be simplified using trigonometric identities to yield

$$p(t) = V_{rms}I_{rms}(\cos(\theta)(1 + \cos(2\omega t)) + \sin(\theta) \sin(2\omega t)) \quad (6)$$

The instantaneous power has a non-zero average and a double-frequency ripple component. We define active power P and reactive power Q as

$$P = V_{rms}I_{rms} \cos(\theta) \quad (7)$$

$$Q = V_{rms}I_{rms} \sin(\theta) \quad (8)$$

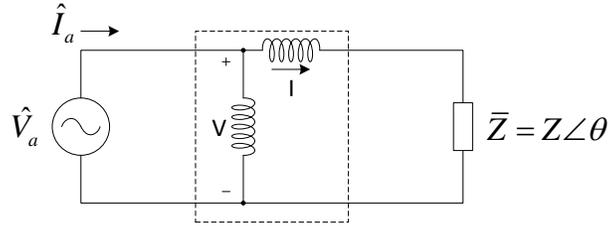


Figure 1: Single-phase power measurement with one wattmeter.

to characterize the total. These may be viewed as the real and imaginary parts of complex power \bar{S} , that is,

$$\bar{S} = P + jQ = V_{rms}I_{rms}\underline{\theta} \quad (9)$$

If the voltage and current are represented as phasors, then

$$\bar{S} = \hat{V}_a \hat{I}_a^* \quad (10)$$

where ‘*’ represents the complex conjugate operator.

Figure 1 shows a single-phase circuit with a wattmeter, represented by a voltage sensor (a high-impedance coil or element) and current sensor (a low-impedance coil or element). A wattmeter is a four-terminal device that simultaneously measures $V_{rms} = |\hat{V}_a|$, $I_{rms} = |\hat{I}_a|$, and P ,

$$P = |\hat{V}_a| |\hat{I}_a| \cos(\angle \hat{V}_a - \angle \hat{I}_a) \quad (11)$$

From this information, we can also find the magnitude of the complex power (typically called *apparent power*, S) and the reactive power.

$$S = |\bar{S}| = V_{rms}I_{rms} \quad (12)$$

$$Q = \sqrt{S^2 - P^2} \quad (13)$$

Modern wattmeters can also perform many internal mathematical operations and report S , Q , and other variables directly.

In a three-phase positive-sequence (abc) system, the line-to-neutral voltages are

$$v_a = \sqrt{2}V_{rms} \cos(\omega t) \quad (14)$$

$$v_b = \sqrt{2}V_{rms} \cos(\omega t - 120^\circ) \quad (15)$$

$$v_c = \sqrt{2}V_{rms} \cos(\omega t + 120^\circ) \quad (16)$$

the line currents are

$$i_a = \sqrt{2}I_{rms} \cos(\omega t - \theta) \quad (17)$$

$$i_b = \sqrt{2}I_{rms} \cos(\omega t - \theta - 120^\circ) \quad (18)$$

$$i_c = \sqrt{2}I_{rms} \cos(\omega t - \theta + 120^\circ) \quad (19)$$

and the instantaneous power is the sum of power on the three phases,

$$p(t) = p_a + p_b + p_c = v_a i_a + v_b i_b + v_c i_c \quad (20)$$

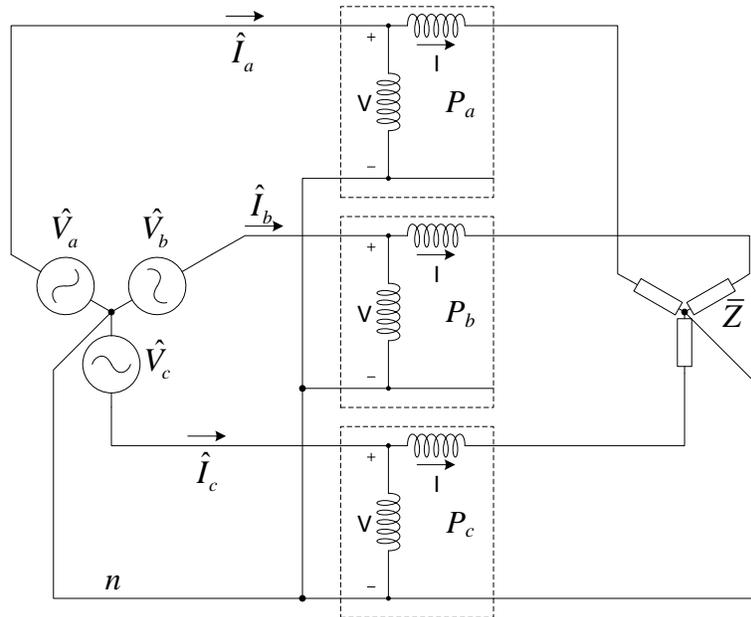


Figure 2: Three-phase power measurement with three-wattmeter method.

Conveniently, the double-frequency ripple terms all cancel—one of the advantages of three-phase power over single-phase power. Just as instantaneous power may be summed, so also with active, reactive, and complex powers.

$$P = P_a + P_b + P_c \quad (21)$$

$$Q = Q_a + Q_b + Q_c \quad (22)$$

$$\bar{S} = \bar{S}_a + \bar{S}_b + \bar{S}_c = P + jQ \quad (23)$$

The naïve approach, then, is the *three-wattmeter method*, illustrated in Figure 2. While this works, it is generally more convenient to use instead the *two-wattmeter method*, illustrated in Figure 3. Notice that the three-wattmeter method references all three meters to the neutral, which may not actually be available. The two-wattmeter method instead references both meters to phase B, which must be available.

In three-phase systems, the specified voltage is typically the *line voltage*, that is, the line-to-line RMS voltage V_{LL} , which is related to the line-to-neutral voltage as

$$V_{LL} = \sqrt{3}V_{rms} \quad (24)$$

Taking phase A line-to-neutral voltage as the angle reference, important phasors are:

$$\hat{V}_a = V_{rms}/0^\circ \quad (25)$$

$$\hat{V}_b = V_{rms}/-120^\circ \quad (26)$$

$$\hat{V}_c = V_{rms}/+120^\circ \quad (27)$$

$$\hat{V}_{ab} = V_{LL}/30^\circ \quad (28)$$

$$\hat{I}_a = I_{rms}/-\theta \quad (29)$$

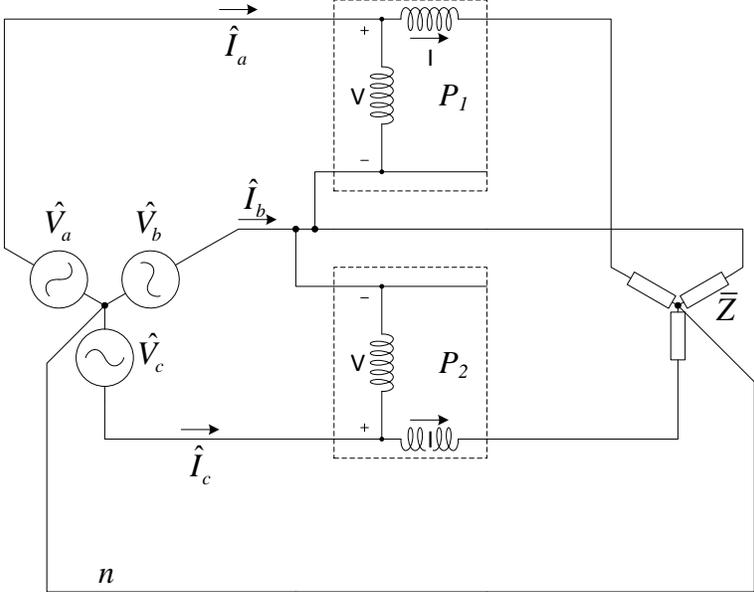


Figure 3: Three-phase power measurement with two-wattmeter method.

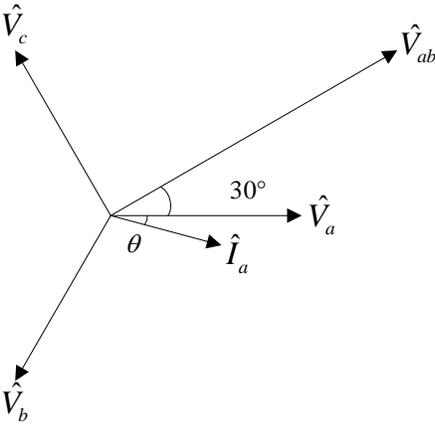


Figure 4: Phasor diagram for typical three-phase system.

These phasors are illustrated in Figure 4.

From the connection diagram in Figure 3, the wattmeters report

$$P_1 = |\hat{V}_{ab}| |\hat{I}_a| \cos(\angle \hat{V}_{ab} - \angle \hat{I}_a) \quad (30)$$

$$P_2 = |\hat{V}_{cb}| |\hat{I}_c| \cos(\angle \hat{V}_{cb} - \angle \hat{I}_a) \quad (31)$$

Using the phasor diagram of Figure 4 and the various relationships,

$$P_1 = V_{LL} I_{rms} \cos(\theta + 30^\circ) \quad (32)$$

$$P_2 = V_{LL} I_{rms} \cos(\theta - 30^\circ) \quad (33)$$

$$P_1 + P_2 = \sqrt{3} V_{LL} I_{rms} \cos(\theta) \quad (34)$$

$$P_2 - P_1 = V_{LL} I_{rms} \sin(\theta) \quad (35)$$

Thus we can find total three-phase active and reactive power using

$$P = P_1 + P_2 \quad (36)$$

$$Q = \sqrt{3}(P_2 - P_1) \quad (37)$$

$$\bar{S} = P + jQ \quad (38)$$

NOTE: If the system is negative sequence (acb), then (37) must be changed to

$$Q = \sqrt{3}(P_1 - P_2) \quad (39)$$

2 Laboratory Software

Consult the lab manual for details on configuring the software. For this experiment, you will primarily use the Yokogawa power meter, in one of the following modes:

- 1P3W: single-phase, for either a single-phase system (channel 1) or a three-phase system treated as three separate phases (all three channels)
- 3P3W: three-phase, three-wire, using channels 1 and 3
- 3P4W: three-phase, four-wire, using all three channels

You will also use the oscilloscope with a current probe and a voltage probe to visualize the voltage and current, and to compute the power.

3 Laboratory Experiment

Throughout this experiment, the load box fans should be turned on. Current should not exceed 3 A.

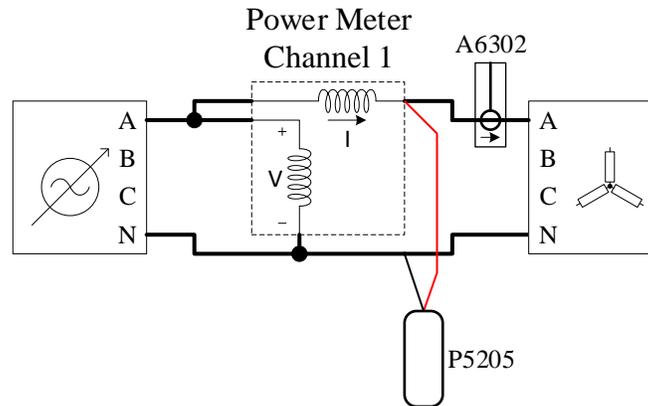


Figure 5: Single-phase power measurement

3.1 Single-Phase

Connect as in Figure 5. Set up the load as RL, as in Figure 8a. **Slowly** increase the voltage until the source reads 100 V, verifying that the voltage and current measured by the power meter are appropriate.

Observe the phase difference between the voltage and current. Observe the pulsating nature of the power waveform, with non-zero average. Record the measured values and save the waveform. Reduce the voltage to zero and turn off the switch.

3.2 Three-Phase Three-Wattmeter

Connect as in Figure 6. Leave the load set as RL type. Follow the standard color code as specified in the lab manual. Again increase the voltage to 100 V.

First, record the measurements with the power meter set to 1P3W mode. In this mode, each channel measures independently. The total power is the sum of the three channel readings.

Next, change the power meter to 3P4W mode. In this mode, the meter performs the summation (Σ). Record the measurements. You will later compare the calculations.

Reduce the voltage to zero and turn off the switch.

3.3 Three-Phase Two-Wattmeter

Connect as in Figure 7, still with an RL load. For this method, you will have a total of ten different measurements to record, all with the source set to 100 V, in two sets of five. For each load setting shown in Figure 8, record:

- Measurements in 1P3W mode, channels 1 and 3
- Measurements in 3P3W mode, the Σ value

Between these two measurements, do not make any other changes. Between **load** setting, reduce the voltage to zero and switch it off. Test with all five load types in Figure 8.

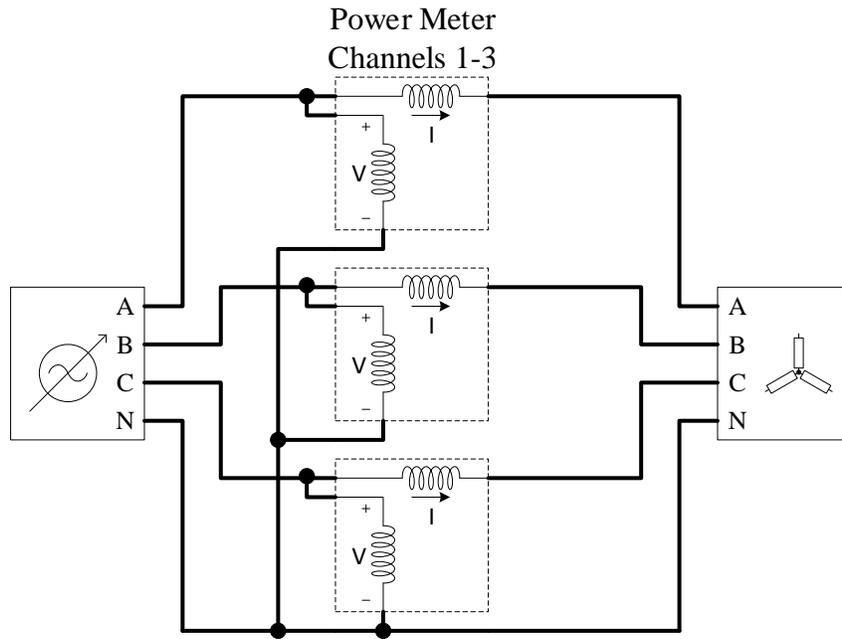


Figure 6: Three-phase power measurement with three-wattmeter method (four-wire).

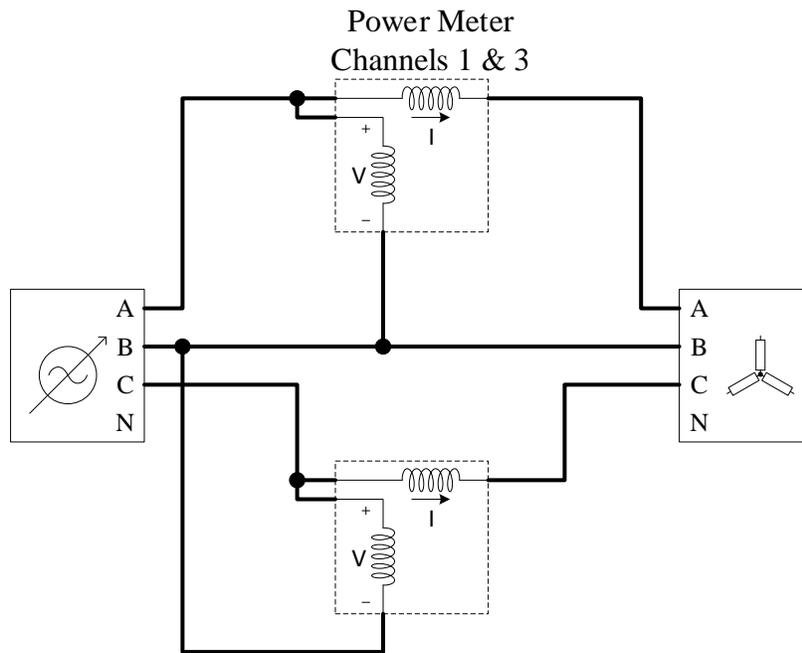
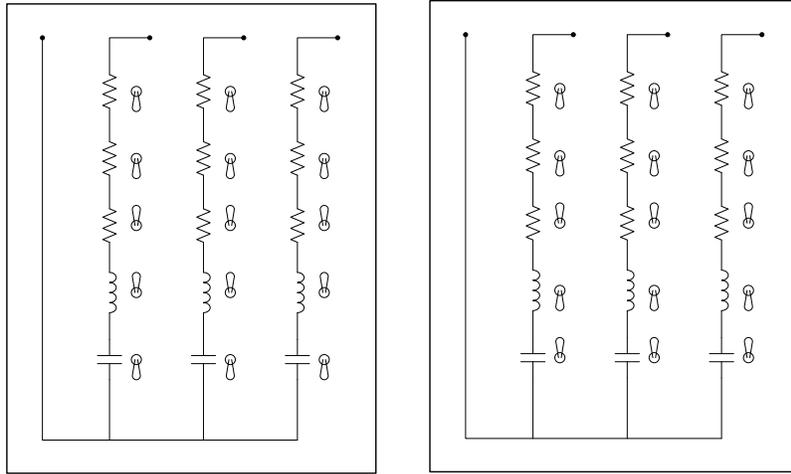
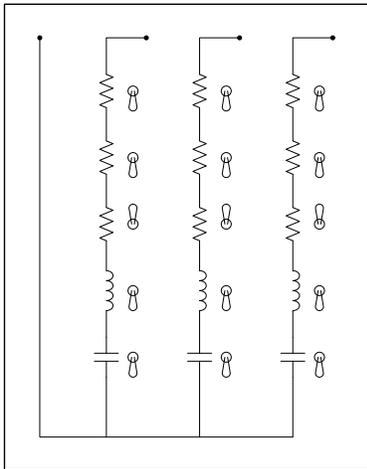


Figure 7: Three-phase power measurement with two-wattmeter method (three-wire).

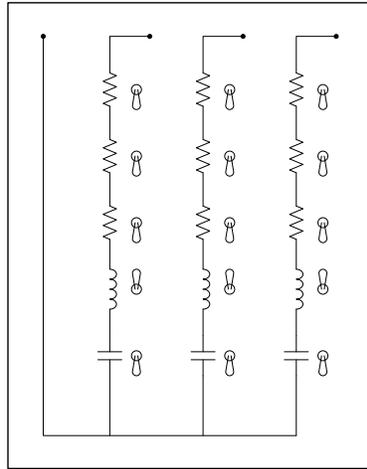


(a) RL load

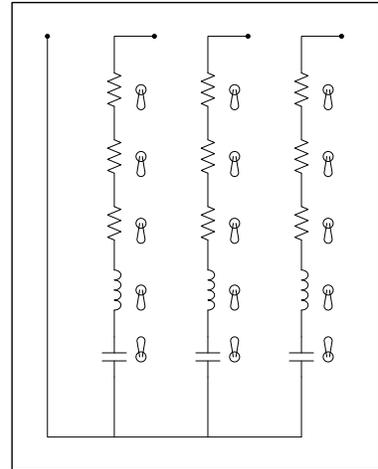
(b) RC load



(c) R load



(d) L load



(e) C load

Figure 8: Load box configurations

4 Calculations and Question

1. For a three-phase system with voltages and currents of

$$v_a = \sqrt{2}V_{\text{rms}} \cos(\omega t)$$

$$v_b = \sqrt{2}V_{\text{rms}} \cos(\omega t - 120^\circ)$$

$$v_c = \sqrt{2}V_{\text{rms}} \cos(\omega t + 120^\circ)$$

$$i_a = \sqrt{2}I_{\text{rms}} \cos(\omega t - \theta)$$

$$i_b = \sqrt{2}I_{\text{rms}} \cos(\omega t - \theta - 120^\circ)$$

$$i_c = \sqrt{2}I_{\text{rms}} \cos(\omega t - \theta + 120^\circ)$$

Derive equations for the instantaneous power for each phase. Using these expressions, show that the total instantaneous power is $P = 3V_{\text{rms}}I_{\text{rms}} \cos \theta$. Discuss how the waveforms from the single-phase and three-wattmeter measurements validate the instantaneous power equations.

2. Show equations for obtaining the average active power P and the average reactive power Q based on the three-wattmeter measurements P_1 , P_2 , and P_3 , as well as the voltage and current measurements V_{l-l} and I_{rms} . With this information, is it possible to determine the sign of the reactive power (and thus the power factor of the load)?
3. Discuss, in generic terms, the effects of load power factor and source phase sequence on wattmeter readings in the two-wattmeter method.
4. For the three-wattmeter method measurements, compare the raw 1P3W results to the Σ value reported by the meter.
5. Using the two-wattmeter method measurements, calculate P , Q , S , θ , and power factor (indicate leading or lagging) for each set of data using only the 1P3W wattmeter readings. Compile these results in a table along with the Σ values reported by the meter. Discuss how the active power, reactive power, power factor, and impedance angle for each case compare to theoretical expectations.
6. From the measurements on the pure elements R, L, and C, calculate the resistance, inductance, and capacitance. Also compute the series resistance for the L and C elements.