EE3501, Experiment 04
Solenoid Simulation

Jonathan Kimball, August 31, 2020

Abstract

This lab uses Simulink to explore the behavior of a solenoid, which is a common electromechanical actuator for linear (translational) motion. Solenoids comprise an electrical input, a magnetic coupling structure, and a mechanical output. Solenoids are widely used to operate valves, switches, etc.

1 Introduction

A typical solenoid is illustrated in Figure 1. The structure has rotational symmetry, i.e., it is essentially cylindrical. A ferromagnetic (steel) outer structure (often termed the yoke) contains a cylindrical coil. A steel plunger is placed along the axis of the coil. Non-magnetic guide rings are used to ensure proper alignment and allow the plunger to slide freely.

When a voltage is applied to the coil, flux φ is induced in the plunger and yoke. As a result, energy is stored in the magnetic field. Force is induced that will move the plunger in a direction that increases inductance and reduces reluctance.

One drawback of a system that relies on reluctance force is that such forces only operate in one direction. Therefore, a spring is included that provides a restoring force, opposite in direction to the electromagnetic force. The equilibrium position depends on the flux in the structure and the force provided by the spring. If an external force f_0 is applied, the equilibrium operating point will change.

2 Mathematical Model

Assuming the steel is infinitely permeable and the coil has finite conductivity, the following relationships may be found. These are usually “good enough” assumptions. Practical systems, though, will often operate where the steel is somewhat magnetically saturated, to reduce cost.

$$\lambda = L(x)i \tag{1}$$

$$v_0 = Ri + \frac{d\lambda}{dt} = Ri + L(x)\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt} \tag{2}$$

$$\frac{di}{dt} = \frac{1}{L(x)}\left(v_0 - Ri - i\frac{dL}{dx}\frac{dx}{dt}\right) \tag{3}$$

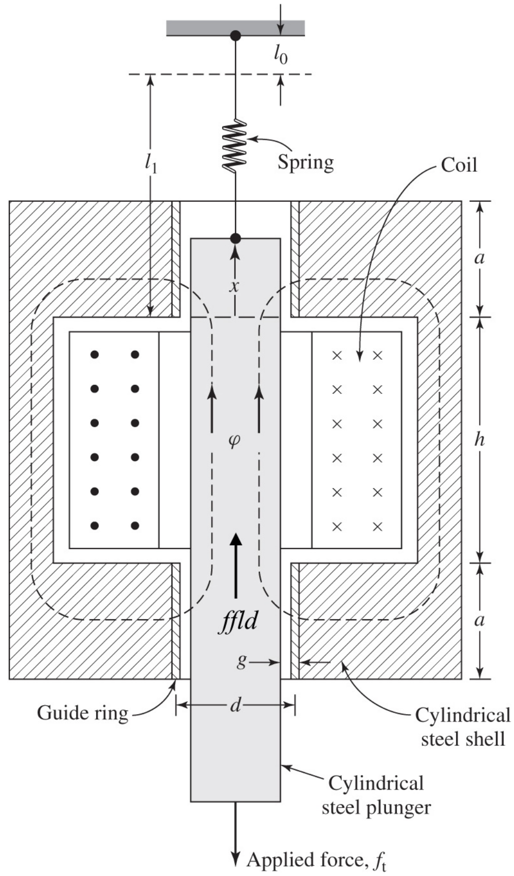

Figure 1: Basic solenoid.

The reluctance varies with position,

$$\begin{aligned}\mathcal{R} &= \frac{g}{\mu_0 \pi x d} + \frac{g}{\mu_0 \pi a d} \\ &= \frac{g}{\mu_0 \pi a d}\left(\frac{a+x}{x}\right)\end{aligned} \tag{4}$$

giving inductance

$$\begin{aligned}L(x) &= \frac{N^2}{\mathcal{R}} = \frac{\mu_0 \pi a d N^2}{g}\left(\frac{x}{a+x}\right) \\ &= L_0\left(\frac{x}{a+x}\right)\end{aligned} \tag{5}$$

where $L_0 = \frac{\mu_0 \pi a d N^2}{g}$. Then the co-energy and force are

$$W'_{fld} = \frac{1}{2} L_0 i^2 \left(\frac{x}{a+x}\right) \tag{6}$$

$$f_{fld} = \frac{\partial W'_{fld}}{\partial x} = \frac{1}{2} L_0 i^2 \left(\frac{a}{(a+x)^2}\right) \tag{7}$$

To complete our analysis of the system, we must apply Newton's laws to the mechanical system.

$$\sum f = ma = m\frac{dv}{dt} \tag{8}$$

$$f_{fld}(i,x) - K(x-x_0) - B\frac{dx}{dt} - f_0 = m\frac{dv}{dt} \tag{9}$$

$$\frac{dv}{dt} = \frac{1}{m}\left(f_{fld}(i,x) - K(x-x_0) - B\frac{dx}{dt} - f_0\right) \tag{10}$$

3 Simulating a Nonlinear Dynamical System

MATLAB®/Simulink® is a development system particularly suited to simulating dynamical systems. A dynamical system is defined by a set of state variables, each of which evolves according to a differential equation. In the present case, the state variables are i, x, and v. The most straightforward way to simulate the behavior of a dynamical system is to compute the derivatives of the state variables, then use integrators to compute the actual state variables. This system has three state equations, repeated here for convenience:

$$\frac{di}{dt} = \frac{1}{L(x)}\left(v_0 - Ri - i\frac{dL(x)}{dx}\frac{dx}{dt}\right) \tag{11}$$

$$\frac{dx}{dt} = v \tag{12}$$

$$\frac{dv}{dt} = \frac{1}{m}\left(f_{fld}(i,x) - K(x-x_0) - B\frac{dx}{dt} - f_0\right) \tag{13}$$

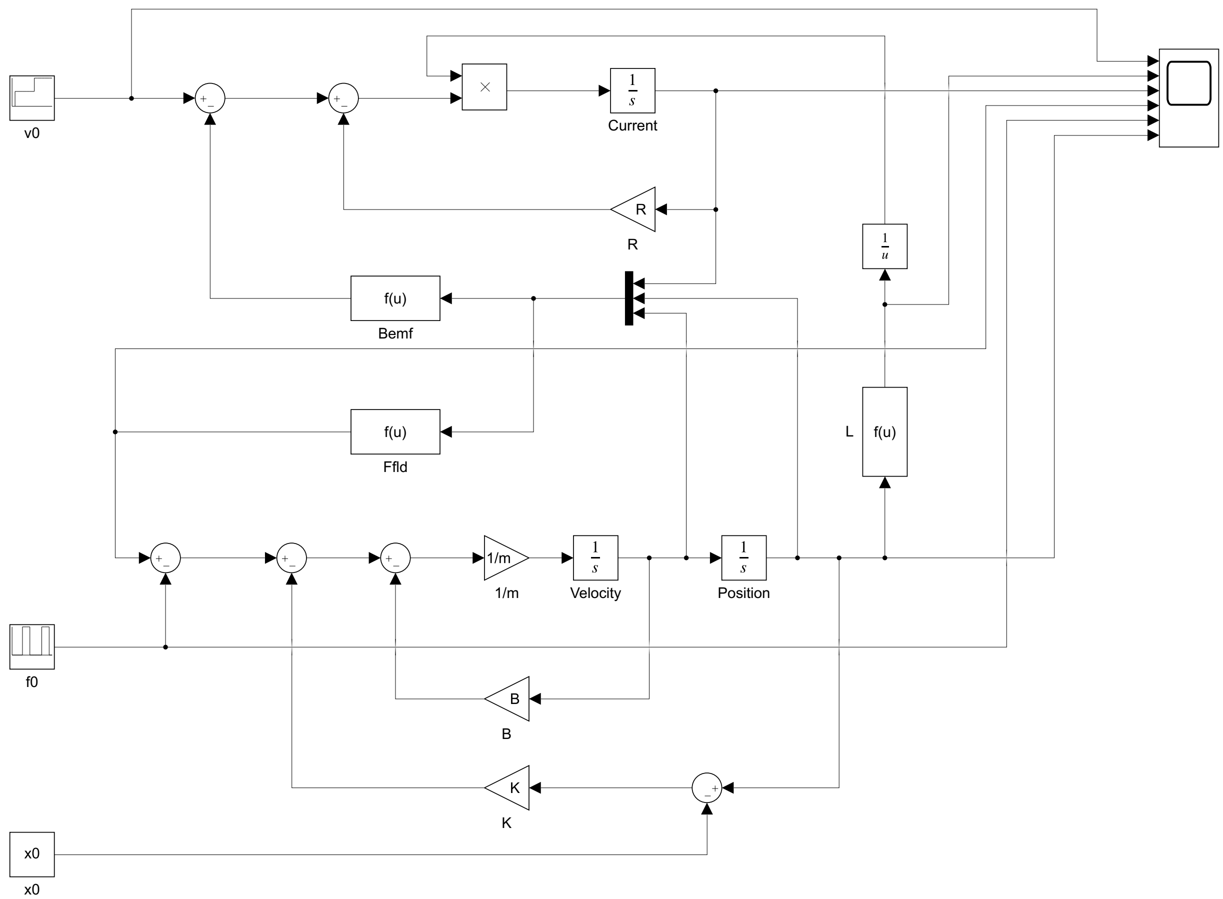

Figure 2: Simulink diagram.

(Time derivatives arise often enough that there is special notation using a dot, e.g. $\dot{x} = v$. When one state variable is i, though, this can become confusing.) So a simulation may be built to compute the right-hand sides of (11)-(13) and integrate them to find the state variables.

The complete system is described by the state variables, the inputs v_0 and f_0, and outputs. In our simulation, outputs will be the three state variables, $L(x)$, and $f_{fld}(i, x)$.

A Simulink diagram is illustrated in Figure 2. Parameters are given in Table 1. Recall that $\mu_0 = 4\pi \times 10^{-7}\mathrm{H} \cdot \mathrm{m}^{-1}$.

4 Laboratory Experiment (Simulation)

Obtain the Simulink file and open it. On the Modeling tab, open the Model Explorer and navigate within the tree to the Model Workspace. You should see all of the parameters listed there. Verify that the parameters match those given in Table 1. Back on the Modeling tab, click on the Model Settings. Ensure that the start time is 0.0, stop time is 1.0, and the solver is Variable-step, auto.

Table 1: System Parameters

Parameter	Value	Unit
a	10×10^{-3}	m
d	12×10^{-3}	m
g	200×10^{-6}	m
N	100	none
R	1.0	Ω
m	0.3	kg
K	2500	N/m
B	2	Ns/m
x_0	2×10^{-3}	m

The system has two inputs: v_0 and f_0. These signals are provided by two "Repeating Sequence Stair" blocks. Open them both and make note of their settings. Then run the simulation and observe the behavior of the system. Save the Scope view by copying it to the clipboard and pasting it into a Word doc, or by exporting it to the workspace and processing it as you desire.

Repeat this process for variations in each of the parameters in Table 1 except for R and x_0. Parameters may be edited within the Model Explorer. Run simulations with each parameter doubled from its default value (only change one variable at a time). Observe the impact on the behavior of the solenoid, both static (equilibrium positions) and dynamic (overshoot, settling time).

5 Calculations and Question

1. For each of the variations in construction parameters (a, d, g, N), compute L_0. What impact does L_0 have on the solenoid's behavior? Are there aspects of the behavior that cannot be correlated with JUST changes to L_0?

2. For each of the variations in mechanical parameters (m, K, B), discuss the impact on static and dynamic behavior.

3. Suppose that your objective is for the solenoid to reach an equilibrium point as quickly as possible, with minimum overshoot, minimum settling time, minimum applied voltage, and minimum change due to external forces. Which of the various parameter combinations is "best"?