EE 216 - Experiment 8
Bode Plots of Frequency Response

Objectives:

To illustrate the relationship between a system frequency response and the frequency response break frequencies, factor powers, and damping ratios by using Bode amplitude and phase plots. Two MATLAB functions are used that compute the frequency response parameters and produce data for the straight line approximation of the Bode amplitude and phase responses.

Theory:

A convenient form of amplitude and phase response plots are Bode plots. For these plots, the amplitude response in dB and the phase response in degrees are plotted on a logarithmic radian/sec. (ω) frequency scale. The use of the dB amplitude scale and the logarithmic frequency scale permits compact representation over wide intervals of amplitude and frequency. It also permits us to plot straight-line approximations to the Bode amplitude and phase responses easily.

The straight-line approximations are achieved by first finding the factor order n and break frequency ω_{bf} parameters for each linear and quadratic factor in the frequency response. In addition, for quadratic factors, a damping ratio ζ where $0 \leq \zeta \leq 1$, is found that identifies the characteristics of the Bode amplitude and phase responses in the neighborhood of the break frequency. Finally, a gain constant C is determined, which may be either positive or negative. The factor order n is positive for a numerator factor and negative for a denominator factor.

The factor order is used to determine the slope of the high frequency straight-line asymptote to the factor's amplitude response plot at high frequencies. It is also used to find the factor's phase response value at high frequencies. The break frequency identifies the boundary between the low-frequency amplitude response asymptote (0 dB) and the high-frequency amplitude straight-line asymptote for the factor. It also identifies the center of the frequency interval over which phase changes appear in the straight-line approximation to the phase response for the factor.

The straight-line approximation to the Bode amplitude response for each factor thus consists of two straight-line asymptotes that have slopes of 0 and $20n$ dB/dec, respectively, and intersect at $\omega = \omega_{bf}$. The straight-line approximation to the Bode phase response for each factor consists of three straight-line segments. The first line is at $0°$ for $\omega \leq 10^{-\gamma}\omega_{bf}$. The second line is at $90n°$ for . $\omega \geq 10^{\gamma}\omega_{bf}$ The third line connects the first and second lines. Note that it passes through $45n°$ at $\omega = \omega_{bf}$. The value of γ is one for a linear factor. For a quadratic factor ,

$$\gamma = \begin{cases} 1.410\zeta - 0.150\zeta^2 & \zeta > 0.2 \\ 1.475\zeta - 0.475\zeta^2 & \zeta > 0.2 \end{cases} \tag{8.1}$$

The straight-line approximation to Bode amplitude and phase response plots are easily constructed from the straight-line approximations for each factor. This is true since only straight

lines need to be added.   It is also easy to determine approximately the effect of adding factors to the frequency response since the straight line approximations of these factors are easily constructed and added.   This is a very useful concept in control system design since the effect of adding compensating systems to the control system is determined easily.   In fact, the nature of the compensating system required can be identified by observing the original system straight-line frequency response approximation and determining break frequencies and slopes that need to be added.

An approximate Bode amplitude response curve can be plotted easily from the straight-line approximations to it.   This is aided by computing one curve deviation point for each break frequency.   The curve deviation from the straight line approximation at the break frequency is $3n\ dB\ and\ 20n\log(2\zeta)$ for linear and quadratic (with $1/\sqrt{2} \le \zeta < 1$) factors, respectively.   If $0 < \zeta < 1/\sqrt{2}$ for a quadratic factor, then a peak appears in the curve.   The peak is located at

$$\omega_p = \omega_{bf}\sqrt{1-2\zeta^2} \tag{8.2}$$

And has a height of

$$M_p = 20n\ \log\left[2\zeta\sqrt{1-\zeta^2}\right]dB \tag{8.3}$$

with respect to the straight line approximation.   It should be noted that the values identified above for total curve deviations from the straight line approximation are accurate for each factor individually but may not give good results for the total curve if factor break frequencies are located close together.   This is because the response factors interact.   That is, if a response factor is not near its asymptote at the break frequency of another response factor then it also causes a non-negligible additive deviation from the straight-line asymptote at this frequency.

An approximate Bode phase response curve cannot be plotted simply from its straight-line approximation by computing a single value for each factor.   The problem is that the response curve relation to the straight-line approximation is more complex for the phase response than it is for the amplitude response.   However in many cases, the straight-line approximation is adequate. Fortunately, if we need the Bode amplitude and phase response curves, they are easily plotted by computer.

MATLAB Functions:

We have developed two MATLAB functions to be used in straight-line Bode plot approximations. The first function is

[rn,rd,imas,rhps,c,bf,ft,dr]=sysdat(n,d)

It computes the break frequencies, **bf**, factor types, **ft**, (linear or quadratic) and damping ratios, **dr**, for the factors of the frequency response defined by the numerator and denominator coefficients **n** and **d**. It also computes the gain constant, **c**. The remainder of the output variables are defined in the help statements contained in the function m-file. These help statements are

function [rn,rd,imas,rhps,c,bf,ft,dr] = sysdat(n,d)
% sysdat.m - computes constant multiplier, break frequencies,
% factor types (1=linear, 2=quadratic, positive
% for numerator factor, negative for denominator
% factor), and quadratic factor damping ratios for
% a transfer function or frequency response with
% numerator coefficients n and denominator
% coefficients d. Coefficient order is highest
% power to zero power of s or jw.
% rn=zeros, rd=poles
% imas=[z p] number of imaginary axis zeros and poles
% rhps=[z p] number of RHP zeros and poles
% c=constant factor (a positive scalar)
% bf=break frequencies (a vector)
%   linear (+,0,-)   quadratic (+,+,+) (LHP,IMA,RHP)
% ft=factor types (a vector)
% linear +1or-1   quadratic +2or-2   + Num, - Denom
% dr=damping ratio (a vector)
% linear or jw dr=10 no damping ratio
% quadratic (+,0,-)   (LHP,IMA,RHP)

The function is also described more completely in Section 2.1.4.1 of the tutorial in Appendix A.

The second MATLAB function

[am,ph]=slbode(w,c,bf,ft,dr)

computes samples for the Bode amplitude and phase response straight-line approximations. These samples are output in the row array variables **am** and **ph**. Inputs are the factor parameters computed by **sysdat** and the frequency row array **w**. There should be an equal number of samples computed in each decade of frequency change. The help statements for this function are

function [am,ph]=slbode(w,c,bf,ft,dr)

% slbode.m - computes the straight lineapproximation to the

% amplitude (am) and phase (ph) bode plots vs. freq. w
% w=frequency scale (rad/sec)
% c=constant factor   (a positive scalar)
% bf=break frequencies (a vector)
%   linear (+,0,-)   quadratic (+,+,+) (LHP IMA RHP)
% ft=factor types (a vector)
% linear +1or-1   quadratic +2or-2   + Num, - Denom
% dr=damping ratio (a vector)
% linear or jw dr=10 no damping ratio
% quadratic (+,0,-)   (LHP,IMA,RHP)

Preliminary:

1.   In Part 1 of this experiment we consider the system represented by the transformed block diagram shown in Figure 8.1.   Find the frequency response for the system.   Note that it is a single quadratic factor.   Find the break frequency, factor exponent, damping ratio, peak height, and peak location for this factor.   Also, compute the gain constant.

2.   In Part 3 of this experiment, we need to compute the damping ratio and break frequency for a quadratic function from its peak height and location.   Obtain an equation for the damping ratio in terms of the peak height from eq. (8.3).   There are two solutions.   The only valid one is the one that is less than $\sqrt{0.5}$.   Also obtain an equation for the break frequency in terms of the peak location and the damping ratio.

Laboratory Procedure:

1.   Find the data for and plot the straight line approximations of the Bode amplitude and phase response for system represented by the transformed block diagram shown in Figure 8.1. Use the two-decade plotting interval $10 \le \omega < 1000 \, \text{rad/sec}$.
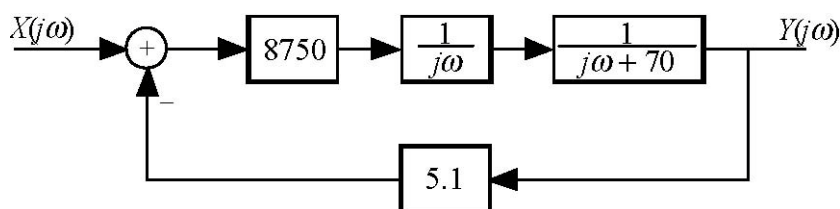

Figure 8.1 Transformed System Block Diagram

Also compute samples for and plot the Bode amplitude and phase responses on the same set of axes as the straight line approximations.   In the previous experiment, we computed samples of the frequency response **H** as a function of the frequency variable **f** (in Hz) by writing the frequency response equation as a mathematical expression statement. We can also compute the samples as a function of the variable **w** in radians/sec using the MATLAB function **H=freqs(n,d,w)** when frequency response numerator and denominator

coefficients **n** and **d** are known.   (If we wanted to compute the samples as a function of **f**, then we could replace **w** with **2 p f**.)

Find the computed peak height and location. Compare the computed break frequency, factor type, damping ratio, peak height, peak location, and gain constant with those determined in the Preliminary. Does the straight-line approximation of the phase response match the phase response well?

2.   An amplifier has the frequency response

$$H\left(j\omega\right) = N(j\omega)/D(j\omega) \tag{8.4}$$

where

$$N(jw) = 4.3200 \text{ x } 10^5 \left(j\omega\right)^4 + 1.0368 \text{ x } 10^{10}(jw)^3 + 6.2208 \text{ x } 10^{13}(j\omega)^2$$
(8.5)

and

$$D(j\omega) = 1.1520(j\omega)^5 + 9.6941 \text{ x } 10^3(j\omega)^4 + 3.5167 \text{ x } 10^7(j\omega)^3$$
$$+1.1113 \text{ x } 10^{11}(j\omega)^2 + 1.6904 \text{ x } 10^{12}(j\omega) + 9.9533 \text{ x } 10^{13} \tag{8.6}$$

a.   Compute and plot the data for the straight-line approximations to the Bode amplitude and phase responses.   Also plot the Bode amplitude and phase responses on the same sets of axes as the straight-line plots.   The frequency interval that you use should encompass all frequencies for which the straight-line phase response approximation is changing.   Do the straight-line approximations give a good picture of the general amplitude and phase response characteristics?

3.   The frequency response of a second-order control system that has only a constant in the numerator and unity DC gain is

$$H(j\omega) = \frac{\omega_{bf}^2}{(j\omega)^2 + 2\zeta\omega_{bf}(j\omega) + \left(\omega_{bf}\right)^2} \tag{8.11}$$

If $\zeta < 1/\sqrt{2}$, then the response of this control system to a unit step command is as shown in Figure 8.2.
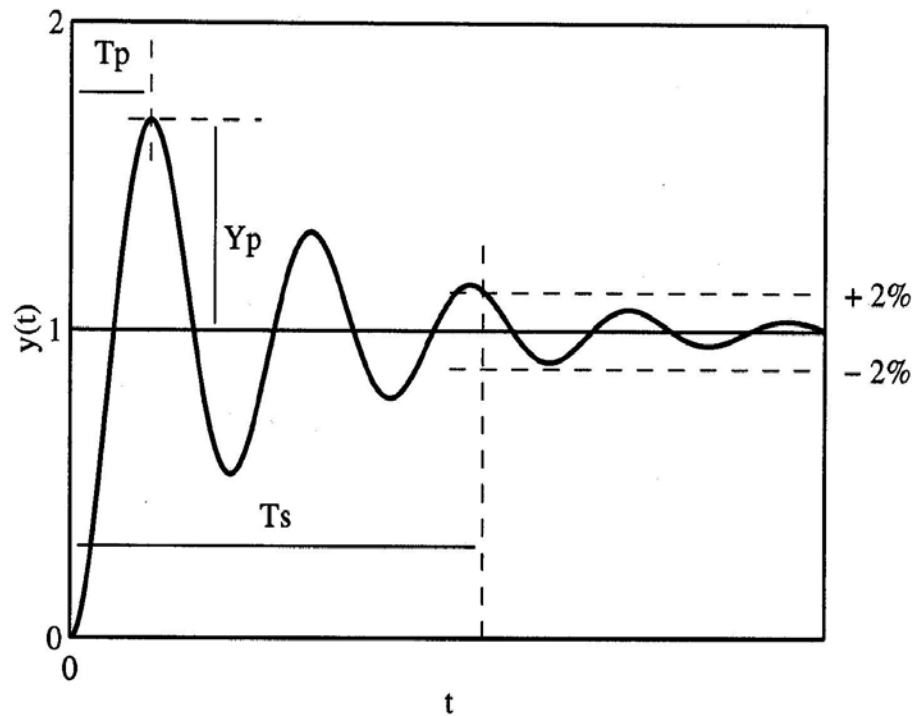
Figure 8.2 Second-Order Control System Unit-Step Response

Three system performance parameters are shown in Figure 8.2. The first is the peak overshoot location T_p, which tells us how quickly the system responds. The second parameter is the settling time T_s, which tells how long it takes for the system to settle to a value close to the desired value. The final parameter is the peak overshoot Y_p. It is of interest since a large overshoot may produce undesirable effects in the controlled quantity. For the second-order system

$$T_p = \pi \left[1 - \zeta^2\right]^{-1/2} \Big/ \omega_{bf} \tag{8.12}$$

and

$$T_s = 4 \Big/ \left(\zeta \omega_{bf}\right) \tag{8.13}$$

The percent overshoot is

$$PO = 100\left(Y_p - 1\right) = \exp\left\{-\zeta\pi(1 - \zeta^2)^{-1/2}\right\} \tag{8.14}$$

Thus we can predict the value of these parameters from the break frequency and damping ratio.

   Many control systems have one denominator quadratic factor in their frequency response that dominates the system's transient response to a step input. Thus, the system behaves approximately as a second-order system. If the quadratic factor has a damping ratio that is less than $1/\sqrt{2}$, the a peak is produced in the frequency response. We can plate the Body amplitude response and measure the height of the peak in dB and the location of the peak in radians/sec. These measurements can then be used in he equation you produced in Part 2 of the Preliminary to compute approximate damping ratio and break frequency values for the quadratic factor. Finally we predict the approximate peak time, settling time, and percent overshoot for the original system using eqs. 8.12), (8.13), and (8.14).

a.   Data file **ex84daa.mat** contains four row arrays. These are **t**, **h**, **w**, and **H**. The variable **h** contains impulse response samples for a control system. The variable **t** is the corresponding time samples. The system frequency response samples are contained in variable **H** corresponding to frequency samples **w** that are equally spaced on a logarithmic scale. Plot the Bode amplitude and phase responses for the system. Identify the dominant peak and compute its height in dB and location in radians/s. Use these values to first compute approximate values for ζ and ω_{bf} (Why are they approximate?) and then compute predictions of T_p, T_s, and PO.

b.   Find the step response by convolving the impulse response and a unit step. Save only the first one-half of the values since the last one-half have serious end effects. Plot the impulse response and the step response. Compute T_p, T_s, and PO for the system from the step response. How well did you predict these values in Part a?