EE 216 - Experiment 6
Spectra of Aperiodic Energy Signals

Objectives:

To find the amplitude and phase spectra of aperiodic energy signals by using the Fourier transform. Also to investigate Fourier transform theorems and computer-evaluated Fourier transform characteristics.

Theory:

The spectrum of the aperiodic energy signal $x(t)$ can be evaluated by using the Fourier transform. It is the frequency domain representation of the signal.

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt \tag{6.1}$$

The amplitude spectrum is $|X(f)|$ and the phase spectrum is $\angle X(f)$. Both are continuous frequency functions. Since the Fourier transform is unique, then the signal can be obtained from the spectrum by using the inverse Fourier transform

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft} df \tag{6.2}$$

Thus, the spectrum characterizes the signal completely.

To evaluate the Fourier transform analytically, we must perform the integration in eq. (6.1). This evaluation is not difficult if the signal is not too complicated. On the other hand, the analytic evaluation of the inverse Fourier transform integral may be quite difficult for even simple signals. However, we can evaluate samples of the spectrum from samples of the signal and vice-versa quite easily with a digital computer.

If the signal is not of finite length, then it must be truncated since the computer can handle only a finite number of samples. The truncation is equivalent to multiplying the signal by the rectangular pulse.

$$w(t) = \Pi\left\{\frac{t - t_a}{t}\right\} \tag{6.3}$$

where truncation starts at $t = t_a - t/2$ and ends at $t = t_a + \tau/2$. The width of the truncated segment is τ. The truncated signal is $x_\tau(t) = x(t)w(t)$. By the convolution theorem and the transform of the rectangular pulse, we know that the spectrum of the truncated signal is

$$X_\tau(f) = X(f) * [\tau sinc(\tau f)e^{-j2\pi ft_a}] \qquad (6.4)$$

We compute samples of this spectrum. Thus we compute a distorted version of the desired spectrum. We say we have truncation distortion. As the truncation width τ increases, the width of the sinc function decreases and less distortion is produced. The sinc function approaches an impulse at $f = 0$ as $\tau \to \infty$. When $\tau = \infty$, $X_\tau(f) = X(f) * \delta(f) = X(f)$ as

expected since the entire signal is being used. Note that the phase spectrum of $X_\tau(f)$ depends

on the locations of the signal portion used. This is again as expected since time-shift corresponds to an additional linear phase term for the transform of $w(t)$. Also note that no truncation distortion exists if the signal is time-limited and the computation interval exceeds the length of the signal.

In most cases, we are interested in real signals. For real signals, $|X(-f)| = |X(f)|$ $and \angle X(-f) = -\angle X(f)$. If in addition, $x(t)$ is even, then $X(f)$ is real and even. In this case, we can plot the total spectrum rather than the amplitude and phase spectra since $X(f)$ is positive or negative but not complex. The positive portions of the spectrum correspond to a phase angle of zero and the negative portions of the spectrum correspond to a phase angle of $\pm\pi$.

The Fourier transform has a number of other properties that are stated as theorems. See the table of these theorems in the text for EE 215/217. We demonstrate a number of these theorems in this experiment.

MATLAB Functions

We have developed two MATLAB functions to be used in Fourier transform evaluation. The first function is

$$[\mathbf{f, X, N, no}] = \mathbf{ctft(t, x, dfm)}$$

It permits you to numerically evaluate eq.(6.1) over the signal truncation interval specified by **t**. This yields samples of the spectrum $X_\tau(f)$ of $x(t)$. As indicated earlier, the truncation

distortion decreases as the width of the truncation interval increases. Remember that no truncation distortion exists if the signal is time limited and the computation interval exceeds the length of the signal. The function **ctft** is briefly described in Section 2.1.3.1. of the tutorial in Appendix A.

The input and output variables and characteristics and constraints of the function are defined in the help statements at the beginning of the function m-file. These statements are reproduced here for your convenience.

function[f, X, N, no] = ctft(t, x, dfm)
% ctft.m -   computes and plots the Fourier transform of signal

```
%                    x(t) extending from t1 to t2 (t1 <= 0 t2 > 0).
%                    dfm = upper limit on transform point spacing
%                    ns = size(x, 2) = number of signal points.
%                    d = signal point spacing.
%                    Transform computed with N points, where N = 2*ns or
%                    N = ceil(1 / (dfm*dt)), whichever is larger.
%                    since N > ns, signal points are zero padded.
```

Note that signal samples must start at $t \le 0$ and end at $t > 0$. This restriction permits script simplification

The number of signal samples that you input is defined to be **ns** in the script. The number of spectrum samples that is computed is **N** where **N** is the larger of **2*ns** or **ceil(1/dfm*dt))** and **dt** and **dfm** are the spacing of the input signal samples and the user specified maximum spacing of the spectrum samples. The additional samples used are all zero samples. Their use does not change the truncated signal. Thus, the spectrum being sampled is unchanged. Only the spectrum sample spacing is changed. **Do not choose dfm and dt so small that the product dfm*dt is very small**. This will produce a large value for **N** with a corresponding long execution time (**N** should probably not exceed 8000, if possible). The spectrum samples are computed over the interval **$-1/(2*dt) < f \le 1/(2*dt)$**. They can be plotted over any interval desired.

The function **ctft** uses an FFT method to compute the coefficients (You will discuss the FFT in EE215). Computation is most rapid if **N** is a power of 2. It is slowest if **N** is a prime number. For most of the computations you will make, this is not a serious consideration. The accuracy of the computed spectrum samples improves as the signal sample spacing decreases. This is particularly true of the samples for the highest frequencies. This inaccuracy is caused by *aliasing*, which is discussed later in your signal and system studies.

The second MATLAB function developed is

$[t, x, N] = ctift(f, X, dtm)$

This function computes samples for the inverse Fourier transform specified by eq. (6.2). If the Fourier transform samples possess complex conjugate symmetry, the inverse transform should be real. In this case, you should take the real part of x before plotting.

This function is briefly described in Section 2.1.3.2 of the tutorial in Appendix A. The help statements contained in the function m-file are

```
function {t, x, N] = ctift(f, X, dtm)
% ctift.m
% function    [t, x, N] = ctift(f, x, dtm)
%                    Computes the inverse Fourier transform of transform X(f)
%                    dtm = upper limit on signal point spacing
%                    ns = size(X) = number of transform points
```

> %   **df = transform point spacing**
> %   **Signal computed with N points, where N = ns or**
> %   **N = ceil(1 / (dtm * df)), whichever is larger.**
> %   **If N.ns, transform points are zero padded.**

Preliminary:

1. Find the analytical spectrum of $x(t)$ in Part 1 of Laboratory Procedure, below.

2. Find the analytical spectrum of $x(t)$ in Part 2, below.

3. Find the analytical spectrum of $x(t)$ and $y(t)$ in Parts 3a and 3b, below.

HINT: Use the Fourier Transform tables in the back of the text, and you may find it helpful to find ways to write these functions so they may be plotted in MATLAB.

Laboratory Procedure:

1.   Compute the Fourier transform of the signal $x(t) = 2sinc(3.2(t-1.5))$ and the inverse Fourier transform of the resulting spectrum. Use $-1 \le t \le 4$ with a spacing of 0.01. Plot the computed amplitude and phase spectra. Also plot the analytically derived amplitude and phase spectra on the same sets of axes. Plot the signal and the signal obtained with the inverse Fourier transform on the same set of axes. Are any discrepancies noted?

2.   We investigate the effect of truncating a signal in this part of the experiment. To do so we use the signal $x(t) = 0.6sinc^2(0.3t)$ for which we know the spectrum.

   a. On one figure, plot the signal from $-15 \le t \le 15$ on the first subplot and plot the analytical spectrum of the signal of the second subplot. Use a sample spacing of 0.05.

   b. Truncate the signal for the interval $-10 \le t \le 10$. Do the following on one figure. Plot the truncated signal from $-15 \le t \le 15$ on the first subplot. On the second subplot, plot the analytical spectrum from part a and the computed spectrum for the truncated interval.

   c. Repeat part b for $-6 \le t \le 6$.

   d. Repeat part b for $-4 \le t \le 4$.

   e. Discuss the effect of signal truncation.

3.   In this part of the experiment, we investigate a couple of Fourier transform theorems, using a signal with a known transform. The signal is $x(t) = 3.6te^{-2.4t}u(t)$.

a. Plot the signal for $-1 \le t \le 5$ using a sample spacing of 0.01. Compute the spectrum using a spectrum sample spacing of 0.02. Plot the computed and known amplitude and phase spectra on the same sets of axes for $-2 \le f \le 2$.

b. Repeat Part a with the signal replaced by the time-shifted signal $y(t) = x(t-0.1)$. In addition, plot the two components of the known phase spectrum (one component is due to the original signal, x(t), and the other is due to the time shift). Make all of the phase plots on one set of axes and note the characteristics of the phase components and compare the computed and known phase characteristics.

4.   The microphone output voltage signal for the words "those young outlaw" spoken by a UMR graduate student is stored in file **tyo.mat**. The signal values are stored in row array **tyo** and the sample spacing (0.000125 s) is stored in variable **T**. The signal length is just over 0.9 s (7301 samples). Plot the signal over the interval $0 \le t \le 1\,s$. Compute samples of the double-sided spectrum for the signal. Then plot the amplitude spectrum for the signal over the interval $0 \le f \le 4\,kHz$.

Listen to the spoken words by using earphones and the MATLAB command **sound(tyo,1/T)**, after loading the **tyo.mat** file. Listen to the words with a smaller sample spacing of 2T/3 and a larger sample spacing of 3T/2. Explain the results observed for the sample spacing changes. Over what range of frequencies does the spectrum contain the most information?