EE 216 - Experiment 4
Amplitude and Phase Spectra Bandwidth

Objectives:

To demonstrate the concepts of amplitude spectrum, phase spectrum, and bandwidth.

Theory:

The frequency domain representation of a signal is given by the frequency spectrum of the signal. The signal spectrum consists of an amplitude spectrum and a phase spectrum. The amplitude spectrum specifies the amplitude of signal components as a function of component frequency. The phase spectrum specifies the phase of signal components as a function of component frequency. This phase is measured with respect to a cosine reference. For example,

$$\begin{aligned} x(t) &= 4\cos[30\pi - \pi/4] - 2\sin[40\pi t + \pi/2] \\ &= 4\cos[2\pi(15)t - \pi/4] - 2\cos[2\pi(20)t] \\ &= 4\cos[2\pi(15)t - \pi/4] + 2\cos[2\pi(20)t + \pi] \end{aligned} \qquad (4.1)$$

has components with amplitudes of 4 and 2 and phases of $-\pi/4$ and π at frequencies of 15 and 20 Hz. The amplitude and phase spectra can be plotted either as single-sided or double-sided. The double-sided spectrum results from the representation of the signal component

$x_1(t) = A_1 \cos\left[2\pi f_1 t + \theta_1\right]$ by

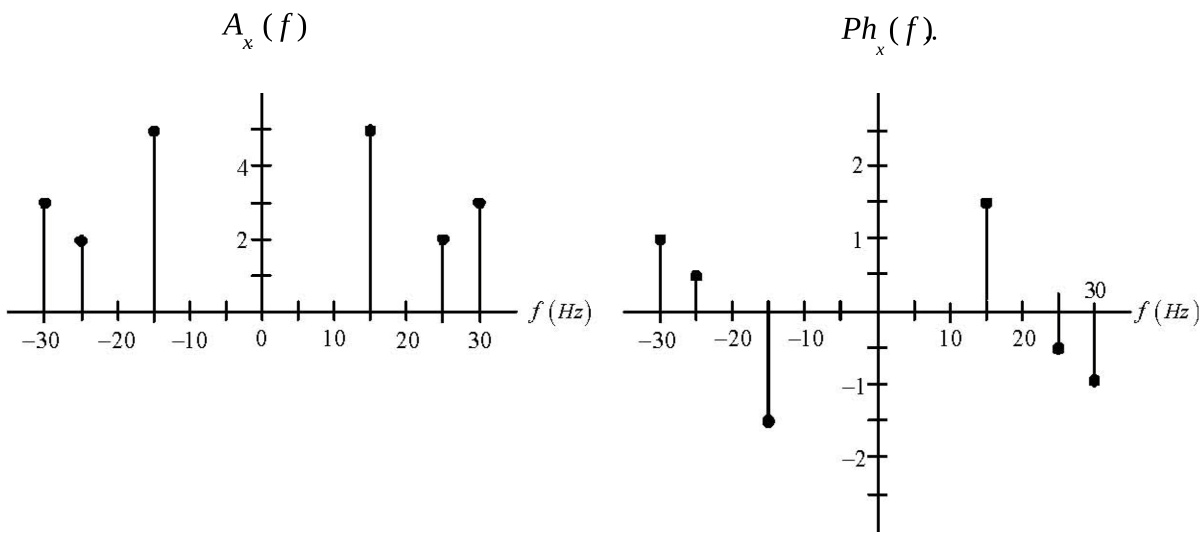
$$x_1(t) = \frac{A_1}{2} e^{j\theta_1} e^{j2\pi f_1 t} + \frac{A_1}{2} e^{-j\theta_1} e^{j2\pi(-f_1)t} \qquad (4.2)$$

The amplitude spectrum contains the value $A_1/2$ at frequencies f_1 and $-f_1$. The phase spectrum contains the values θ_1 and $-\theta_1$ at frequencies f_1 and $-f_1$, respectively. Thus, the double-sided spectra for $f > 0$ looks like the single-sided spectra except that amplitude values for $f > 0$ are one-half as large. If a DC component exists (that is, a component at $f = 0$) then the amplitude spectrum value for this component is the same for both the single-sided and double-sided spectrum.

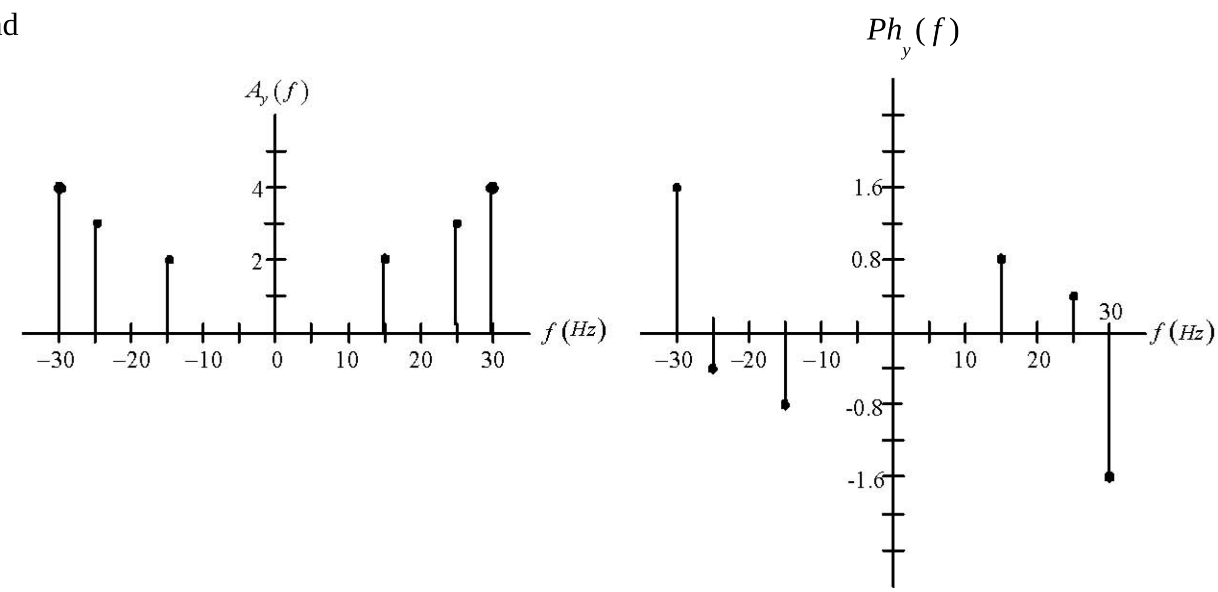
The signal bandwidth, B, is defined to be the difference between the maximum and minimum positive frequencies for which the amplitude spectrum $A(f)$ is greater than or equal to a times the maximum amplitude spectrum value $A(f)_{Max}$. Note that this definition works equally well for single-sided and double-sided spectra. The significance value α is a selected constant. The half-power or 3-dB bandwidth is defined when $\alpha = 1/\sqrt{2} = 0.707$. Another value commonly used for the significance factor is $\alpha = 0.1$.

Laboratory Procedure:

1. The double-sided amplitude and phase spectra for *x*(*t*) and *y*(*t*) are



and



respectively.

a. Plot *x*(*t*) and *y*(*t*) for $-0.2 \le t \le 0.2$ Note that the two signals contain the same set of frequencies and yet look significantly different.

b. A signal looks significantly different than another signal even when it contains the same frequencies and amplitude if the component phases are different. Show this by plotting *z*(*t*) where $A_2(f) = A_x(f)$ *and* $Ph_2(f) = Ph_y(f)$ and visually compare the plots of

$z(t)$ and $x(t)$. Note that a comparison of $z(t)$ and $x(t)$ shows the effect of changing the signal component phases only.

2. Plot the signal

$$x(t) = -1.3 - 8.4\cos(1.5\pi t - 0.45) + 4.2\sin(23t + 2.8) - 4.8\sin(12\pi t - 1.3)$$

for $-0.5 \le t \le 0.5$.
Plot the single-sided and the double-sided amplitude and phase spectra for this signal.
Hint: Use the **stem** plotting command, or modify the plot command to use points instead of lines.

3. Consider the system represented by the equations given in Experiment 3 Part 4.

   a. Reuse the script from Experiment 3 Part 4, where the samples of an impulse response were convolved with samples of an input to produce samples of an output. Given the input signal
   $x(t) = -3.9\cos(0.2\pi t - 1.5) + 3.75\cos(0.5\pi t - 0.6) + 0.5\cos(1.2\pi t + 0.2)$,
   compute samples of the output signal for each of the three *components* of the input signal. Compute the output samples over the interval $-22 \le t \le 10s$ and then plot each input component and its output for $-10 \le t \le 10s$. Use one set of axes for each input-output component pair. The first 12s of the output are not plotted to eliminate the initial time end effect.

   b. Use the principle of superposition to find samples of the output signal corresponding to this input signal and plot the input and output signal on one set of axes.

   c. Plot the double-sided amplitude spectrum for the input and output signals. (**Hint:** use the **max** command to find the amplitude of each component of the output signal for $t \ge 0$.)

   d. What is the significant bandwidth of the input and output signals if we use a significance factor of 0.2?