Control Systems Laboratory (EE 3321) — Experiment 8
QUARC Software, Couple Tank Hardware Integration, Single Tank Level Control

I. Overview of Experimental Procedure

For this experiment, the student will make a Simulink model using QUARC blocks to control the water level in tank1, using the outflow from the pump which composes of DC motor. Using the principles learned from that task, the student will then design a model for level controller. Additionally, the student will analyze the response of the system to determine the gain and time constants for the equipment used. This will allow the student to create a detailed transfer function which models the system, and the student will be able to compare the theoretical and actual responses in real time.

II. Background on the QUARC Software and Hardware

The QUARC software is used with Simulink to interact with the hardware of the couple tank system. QUARC is used to drive the pump and read the data from pressure sensors. The basic steps to create a Simulink model with QUARC in order to interact with the Coupled Tank System hardware are:

- Make a Simulink model that interacts with your installed data acquisition device using blocks from the QUARC Targets library.
- Build the real-time code.
- Execute the code.

Type doc quarc in the MATLAB Command Window to access QUARC documentation and demos.

A schematic of the Coupled-Tank plant is represented in Figure 2.1, below. As illustrated in Figure 2.1, the positive direction of vertical level displacement is upwards, with the origin at the bottom of each tank (i.e. corresponding to an empty tank).

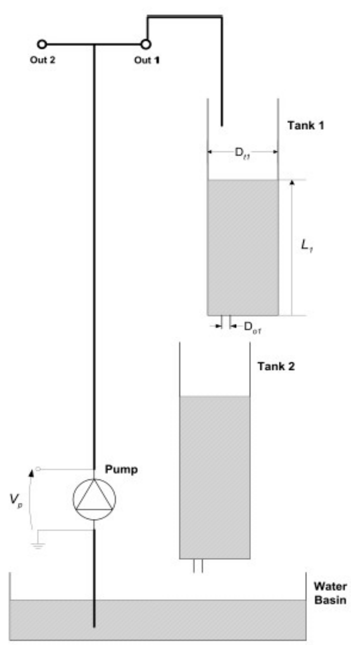


Figure 7.1: Schematic of Coupled Tank.

The system's two water tanks are made out of Plexiglas tubes of uniform cross section. The Coupled-Tank pump is a gear pump composed of a DC motor rated for 12 V continuous and 22 V peak with heat radiating fins.

Each tank's actual liquid level is measured through a pressure sensor. Such a level sensor is located at the bottom of each tank and provides linear level readings over the complete liquid vertical level. In other words, the sensor output voltage increases proportionally to the applied pressure. Its output measurement is processed through a signal conditioning board and made available as 0 to 5V DC signal. Moreover, calibration of each pressure sensor's offset and gain potentiometers is required to keep level measurements consistent with the type of liquid used in the coupled-tank experiment.

III. Nonlinear Equation of Motion (EOM)

In order to derive the mathematical model of your Coupled-Tank system in Figure 7.1, it is reminded that the pump feeds into Tank 1 and that Tank 2 is not considered at all. Therefore, the input to the process is the voltage to the pump *VP* and its output is the water level in tank 1, $L1$, (i.e. top tank).

The purpose of the present modelling session is to provide you with the system's open-loop transfer function, $G1(s)$, which in turn will be used to design an appropriate level controller. The obtained Equation of Motion, EOM, should be a function of the system's input and output, as previously defined.

Therefore, you should express the resulting EOM under the following format:

$$\frac{\partial L_1}{\partial t} = f(L_1, V_p) \tag{7.1}$$

Where *f* denotes a function.

In deriving the Tank 1 EOM the mass balance principle can be applied to the water level in tank 1, i.e..

$$A_{t1}\frac{\partial L_1}{\partial t} = F_{i1} - F_{o1} \tag{7.2}$$

Where A_{t1} is the area of Tank1. F_{i1} and F_{o1} are the inflow rate and outflow rate, respectively. The volumetric inflow rate to tank 1 is assumed to be directly proportional to the applied pump voltage, such that:

$$F_{i1} = K_p V_p \tag{7.3}$$

Applying Bernoulli's equation for small orifices, the outflow velocity from tank 1, $vo1$, can be expressed by the following relationship:

$$v_{o1} = \sqrt{2gL_1} \tag{7.4}$$

IV. EOM Linearization and Transfer Function

In order to design and implement a linear level controller for the tank 1 system, the open-loop Laplace transfer function should be derived. However by definition, such a transfer function can only represent the system's dynamics from a linear differential equation. Therefore, the nonlinear EOM of tank 1 should be linearized around a quiescent point of operation. By definition, static equilibrium at a nominal operating point (Vp0;L10) is characterized by the Tank 1 level being at a constant position L10 due to a constant water flow generated by constant pump voltage Vp0.

In the case of the water level in tank 1, the operating range corresponds to small departure heights, L11, and small departure voltages, Vp1, from the desired equilibrium point (Vp0;L10). Therefore, L1 and Vp can be expressed as the sum of two quantities, as shown below:

$$L_1 = L_{10} + L_{11}, \qquad V_p = V_{p0} + V_{p1} \tag{7.5}$$

The obtained linearized EOM should be a function of the system's small deviations about its equilibrium point (Vp0;L10). Therefore, one should express the resulting linear EOM under the following format:

$$\frac{\partial}{\partial t} L_{11} = f(L_{11}, V_{p1}) \tag{7.6}$$

Where *f* denotes a function.

Transfer Function

From the linear equation of motion, the system's open-loop transfer function in the Laplace domain can be defined by the following relationship:

$$G_1(s) = \frac{L_{11}(s)}{V_{p1}(s)|} \tag{7.7}$$

The desired open-loop transfer function for the Coupled-Tank's tank 1 system is the following:

$$G_1(s) = \frac{K_{dc1}}{\tau_1 s + 1} \tag{7.8}$$

Where K_{dc1} is the open-loop transfer function DC gain, and τ_1 is the time constant.

As a remark, it is obvious that linearized models, such as the Coupled-Tank tank 1's voltage-to-level transfer function, are only approximate models. Therefore, they should be treated as such and used with appropriate caution that is to say within the valid operating range and/or conditions. However for the scope of this lab, Equation 2.5 is assumed valid over the pump voltage and tank 1 water level entire operating range, V_{p_peak} and L_{1_max}, respectively.

V. TANK 1 LEVEL CONTROL

For zero steady-state error, tank 1 water level is controlled by means of a Proportional-plus-Integral (PI) closed-loop scheme with the addition of a feedforward action, as illustrated in Figure 7.2, below, the voltage feedforward action is characterized by:

$$V_{p\_ff} = K_{ff\_1}\sqrt{L_{r\_1}} \tag{7.9}$$

and

$$V_p = V_{p1} + V_{p\_ff} \tag{7.10}$$

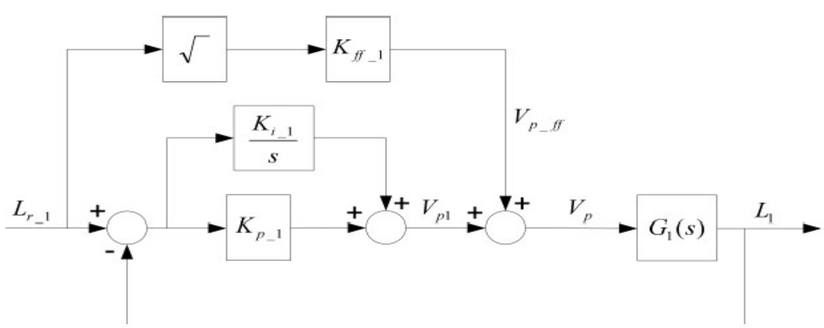


Figure 7.2 Tank 1 Water Level PI-plus-Feedforward Control Loop

The open-loop transfer function $G_1(s)$ takes into account the dynamics of the tank 1 water level loop, as characterized by Equation 7.7. However, due to the presence of the feedforward loop, G1(s) can also be written as follows:

$$G_1(s) = \frac{L_1(s)}{V_{p1}(s)} \tag{7.11}$$

VI. Second-Order Response

The block diagram shown in Figure 3.2 is a general unity feedback system with compensator, i.e., controller C(s) and a transfer function representing the plant, P(s). The measured output, Y (s), is supposed to track the reference signal R(s) and the tracking has to match to certain desired specifications.

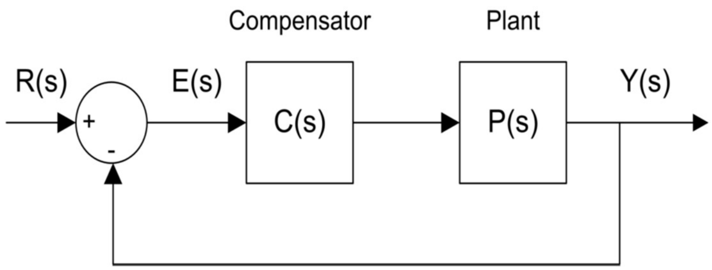


Figure 7.3: Unity feedback system.

The output of this system can be written as:

$$Y(s) = C(s)\,P(s)\,\left(R(s) - Y(s)\right)$$

By solving for Y (s), we can find the closed-loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{C(s)\,P(s)}{1 + C(s)\,P(s)}$$

The input-output relation in the time-domain for a proportional-integral (PI) controller is

$$u = K_p(r-y) + \frac{K_i(r-y)}{s} \tag{7.12}$$

where *Kp* is the proportional gain and *Ki* is the integral gain.

In fact, when a first order system is placed in series with PI compensator in the feedback loop as in Figure 3.2, the resulting closed-loop transfer function can be expressed as:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\,\omega_n\,s + \omega_n^2} \tag{7.13}$$

where ω_n is the natural frequency and ζ is the damping ratio. This is called the standard second-order transfer function. Its response properties depend on the values of ω_n and ζ .

Peak Time and Overshoot

Consider a second-order system as shown in Equation 3.5 subjected to a step input given by

$$R(s) = \frac{R_0}{s} \tag{7.14}$$

with a step amplitude of Ro = 1.5. The system response to this input is shown in Figure 3.3, where the red trace is the response (output), y(t), and the blue trace is the step input r(t).

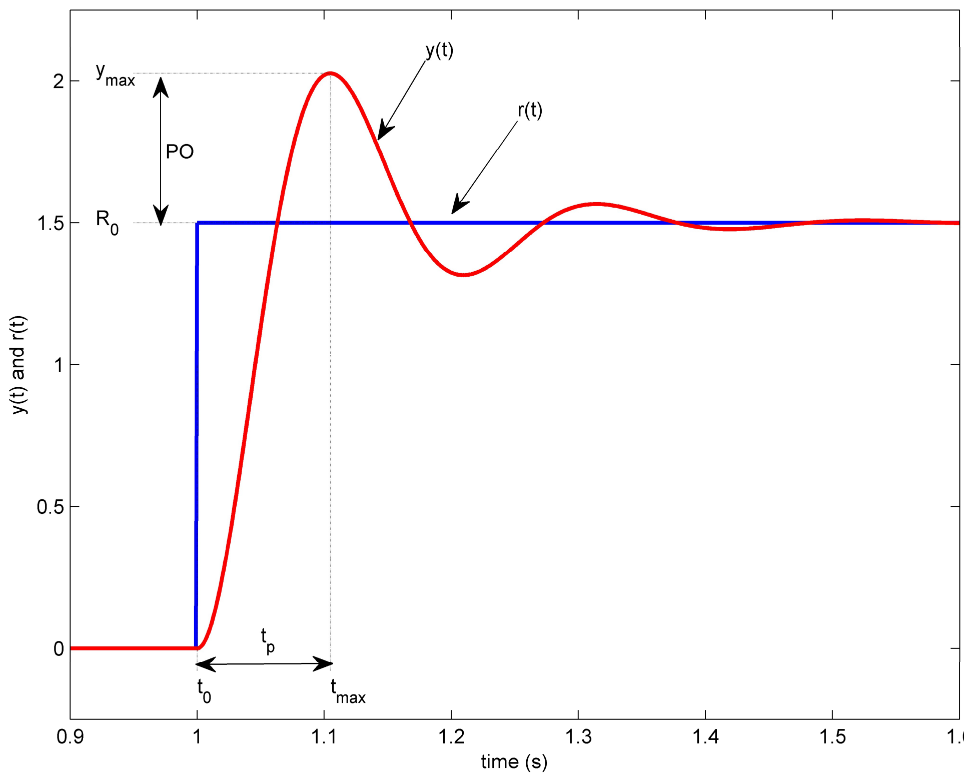


Figure 7.4: Standard second-order step response.

The maximum value of the response is denoted by the variable *ymax* and it occurs at a time *tmax*. For a response similar to Figure 3.3, the percent overshoot is found using

$$PO = \frac{100\,(y_{max} - R_0)}{R_0} \tag{7.15}$$

From the initial step time, t_0 , the time it takes for the response to reach its maximum value is

$$t_p = t_{max} - t_0 \tag{7.16}$$

This is called the peak time of the system.

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the equation

$$PO = 100\,e^{\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)} \tag{7.17}$$

The peak time depends on both the damping ratio and natural frequency of the system and it can be derived as

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{7.18}$$

Tank 1 level response 2% Settling Time can be expressed as follows:

$$t_s = \frac{4}{\zeta\omega} \tag{7.19}$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

VII. Hardware Connections

Below figure sums up the electrical connections necessary to run the Coupled-Tank system.

Cable #	From	To	Signal
1	Analog Output **AO #0**	Amplifier **"Command"** connector	Control signal to the amplifier.
2	Amplifier **To Load** connector	Coupled-Tank's "Pump Connector" connector	Power leads to the gear pump.
3	Amplifier **"To ADC"**	Data Acquisition (DAQ) Device: 1. S1 (yellow) to Analog Input **AI #0** 2. S2 (white) to Analog Input **AI #1**	Tank 1 and tank 2 level feedback signals to the Data Acquisition (DAQ) Device, through the amplifier.
4	Coupled-Tank's "Pressure Sensors Connector" #1	Amplifier **"S1 & S2"**	Liquid level feedback signal to the amplifier.
5	Power Supply Outlet #1	Amplifier Power Socket	Amplifier Power Supply.

Table 7.1: Coupled-Tank Wiring Summary

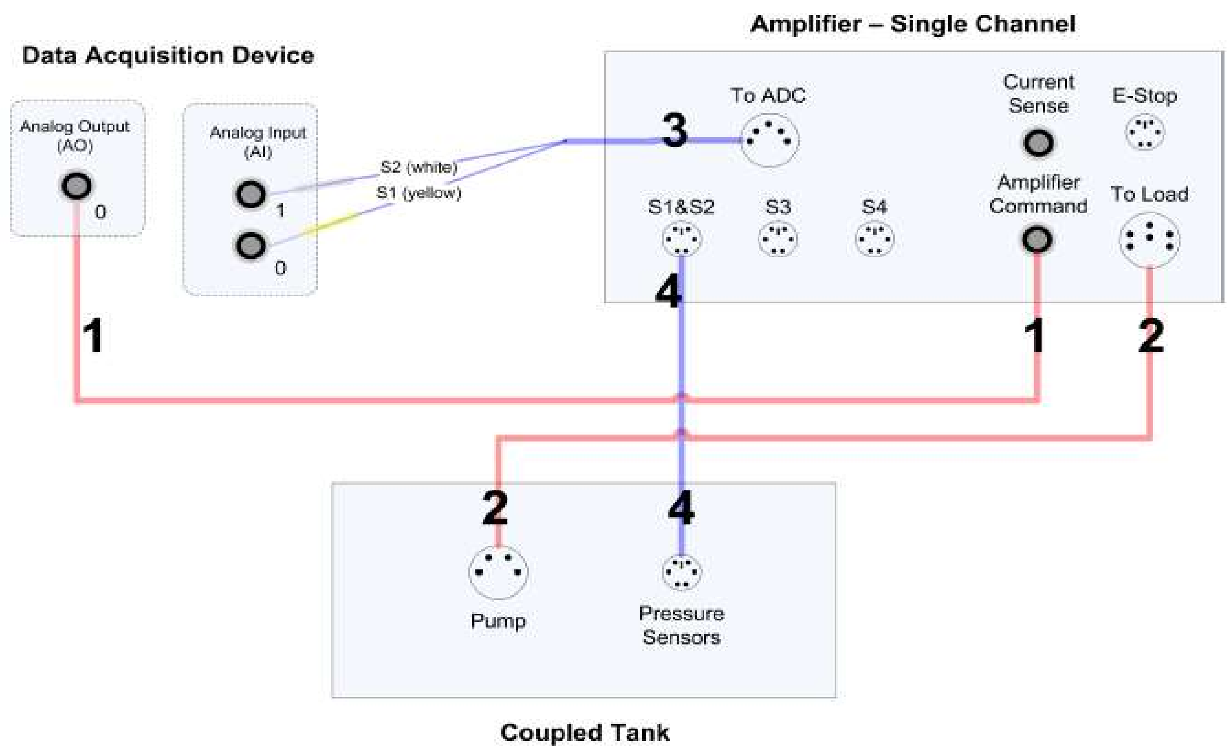


Figure 7.5: Coupled Tank Wiring Diagram

VIII. Configuring a Simulink Model for the Coupled tank

The following steps should be used to build a Simulink model that will interface with the Coupled Tank using QUARC:

- Make sure the Coupled Tank System is connected to the PC through the USB port, and that it is powered on (the Power LED should be lit).
- Open MATLAB Simulink, and create a new Simulink diagram.
- Open the Simulink Library Browser, and expand the QUARC Targets item. From the **Data Acquisition >> Generic >> Configuration** folder, choose the **HIL Initialize** block. Drag this block into the Simulink model. This block is used to configure your data acquisition device. Use the image below for reference.

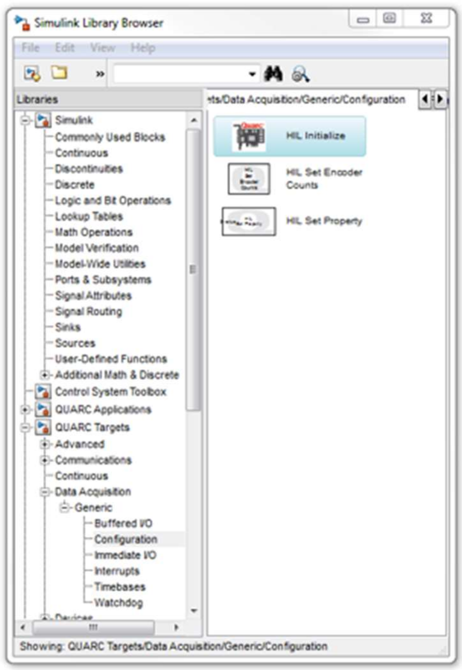


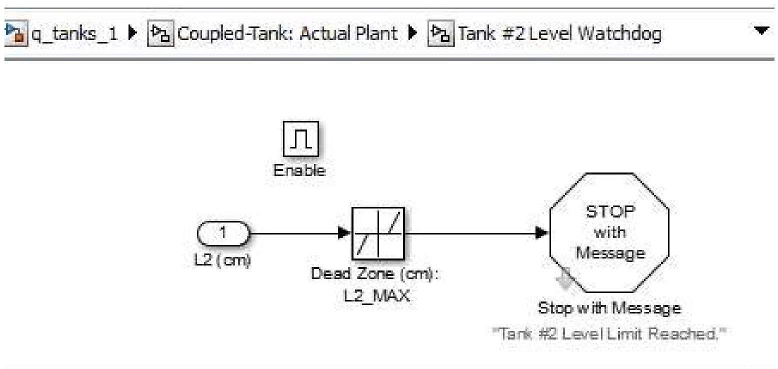
Figure7.6: QUARC targets in SIMULINK library browser

- Double-click on the HIL Initialize block. In the Board type field, select **q2_usb**.
- Use the **QUARC >> Set Default Options** item to set the correct Real-Time Workshop parameters and setup the Simulink model for external use (as opposed to the simulation mode).
- Select **QUARC >> Build**. Various lines in the MATLAB Command Window should be displayed as the model is being compiled. This creates a QUARC executable (.exe) file, which we will commonly refer to as a QUARC controller.
- Run the QUARC controller. To do this, go to the Simulink model toolbar and click on the **Connect to Target** icon and then the **Run** icon. You can also select **QUARC >> Start** to run the code. The Power LED on the DAQ should be blinking.
- If you have successfully ran the QUARC controller without any errors, you can stop the code by clicking the **Stop** icon in the toolbar, or by selecting **QUARC >> Stop**.

IX. Reading the Sensors

The following steps should be used to read the level sensors:

- Using the Simulink model created in the last section, add the **HIL Read Analog Timebase** block from the **QUARC Targets >> Data Acquisition >> Generic >> Timebases** folder in the Library Browser.
- Connect the HIL Read Analog to a Gain block, the gain block is the sensor sensitivity value which converts the sensor outputs to centimeters (6.0241cm/V for coupled tank sensors); connect the output of the Gain block to a Display block.
- Design a first-order low-pass filter and connect it in series to the gain block, this smoothens the jittery sensor data.
- Attach a watchdog subsystem to check the maximum level of tank1. This should auto stop the simulation if the level on tank1 reaches L1_Max (Maximum level for tank 1- 30cms), as shown in figure below.



X. Driving the Pump

The following steps should be used to drive the pump:

- Using the same Simulink model from the previous section, add the **HIL Write Analog** block from the **QUARC Targets >> Data Acquisition >> Generic >> Immediate I/O** folder in the Library Browser. This block is used to output a signal from analog output channel #0 on the data acquisition device to the on-board PWM amplifier, which drives the pump motor.
- Add a Constant block (Amplifier Gain) to the input voltage port.
- Include DAC Limit block in series. This Saturation block imposes upper and lower limits on the input signal to prevent damage to the motor.
- Add a Gain block in parallel to HIL Write Analog and join it to an out port to confirm the measurement.
- Build and run the QUARC controller. You should see the motor pumping the water to tanks
- Stop the QUARC controller.

XI. Tank 1 Level Control Simulation

A control is designed to regulate the water level (or height) of tank #1 using the pump voltage. The control is based on a Proportional-Integral-Feedforward scheme (PI-FF). Given a ± 1 cm square wave level setpoint (about the operating point), the level in tank 1 should satisfy the following design performance requirements:

1. Operating level in tank 1 at 15 cm: $L10$ = 15 cm.
2. Percent overshoot less than 10%: $PO1$ _ 11 %.
3. 2% settling time less than 5 seconds: ts_1 _ 5:0 s.

4. No steady-state error: *ess* = 0 cm.

The following steps should be used for level control simulation:

The *s_tanks_1* Simulink diagram shown in Figure 7.7 is used to perform tank 1 level control simulation exercises in this laboratory.

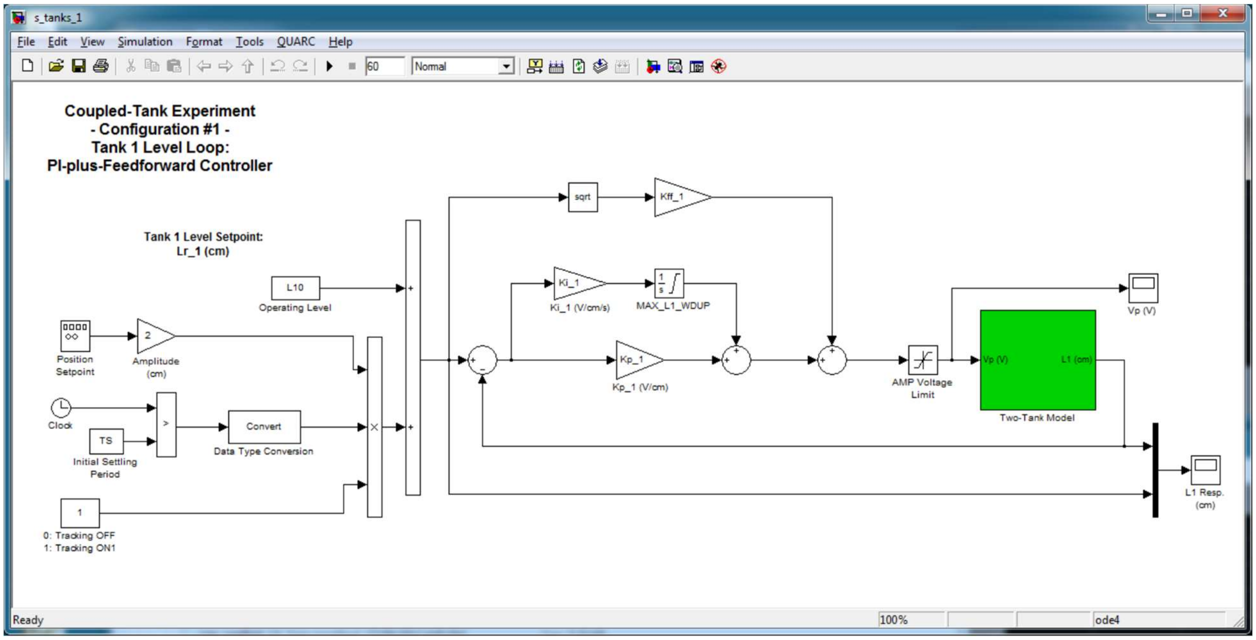


Figure 7.7: Simulink model used to simulate PI-FF control on Coupled Tanks system.

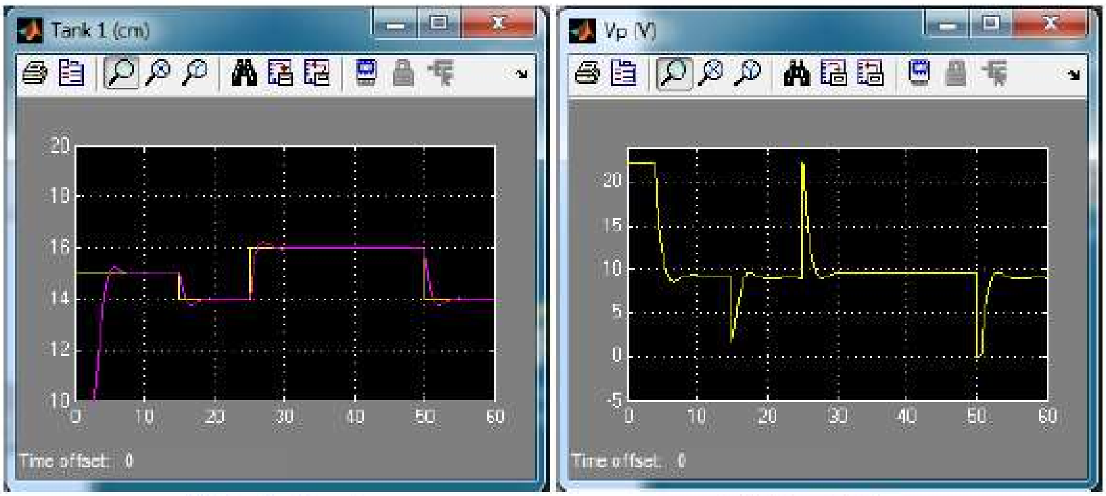
Follow this procedure:

1. Enter the proportional, integral, and feedforward gain control gains found in Matlab as *Kp_1*, *Ki_1*, and *Kff_1*.
2. To generate a step reference, go to the Signal Generator block and set it to the following:
   - Signal type = *square*
   - Amplitude = 1
   - Frequency = 0.02 Hz
3. Set the *Amplitude (cm)* gain block to 1 to generate a square wave goes between ± 1 cm.
4. Coupled-Tank Non-Linear Model subsystem has to be created based on its mathematical model (Block shown in green).
5. Open the pump voltage *Vp (V)* and tank 1 level response *Tank 1(cm)* scopes.
6. By default, there should be anti-windup on the Integrator block (i.e., just use the default Integrator block).
7. Start the simulation. By default, the simulation runs for 60 seconds. The scopes should be displaying responses similar to Figure 3.5. Note that in the *Tank 1 (cm)* scope, the yellow trace is the desired level while the purple trace is the simulated level.
8. Generate a MATLAB figure showing the Simulated Tank 1 response and the pump voltage.

Data Saving:

After each simulation run, each scope automatically saves their response to a variable in the Matlab workspace. The Vp (V) scope saves its response to the variable called data_Vp and the Tank 1 (cm) scope saves its data to the data_L1 variable.

- The data_L1 variable has the following structure: data_L1(:,1) is the time vector, data_L1(:,2) is the set point, and data_L1(:,3) is the simulated level.
- For the data_Vp variable, data_Vp(:,1) is the time and data_Vp(:,2) is the simulated pump voltage.



(a) Tank 1 Level (b) Pump Voltage

Figure 7.8: Simulated closed-loop control response

9. Assess the actual performance of the level response and compare it to the design requirements. Measure your response actual percent overshoot and settling time. Are the design specifications satisfied? Explain. If your level response does not meet the desired design specifications, review your PI-plus-Feedforward gain calculations and/or alter the closed-loop pole locations until they do.

   Hint: Use the graph cursors in the Measure tab to take measurements.

X. Tank 1 Level Control Implementation

The **q_tanks_1** Simulink diagram shown in Figure 3.6 is used to perform the tank 1 level control exercises in this laboratory. The Coupled Tanks subsystem contains QUARC blocks that interface with the pump and pressure sensors of the Coupled Tanks system.

Note that a first-order low-pass filter with a cut-off frequency of 2.5 Hz is added to the output signal of the tank 1 level pressure sensor. This filter is necessary to attenuate the high-frequency noise content of the level measurement. Such a measurement noise is mostly created by the sensor's environment consisting of turbulent flow and circulating air bubbles. Although introducing a short delay in the signals, low-pass filtering allows for higher controller gains in the closed-loop system, and therefore for higher performance. Moreover, as a safety watchdog, the controller will stop if the water level in either tank 1 or tank 2 goes beyond 27 cm.

Experimental Setup

The q_tanks_1 Simulink diagram shown in Figure 3.6 will be used to run the PI+FF level control on the actual Coupled Tanks system.

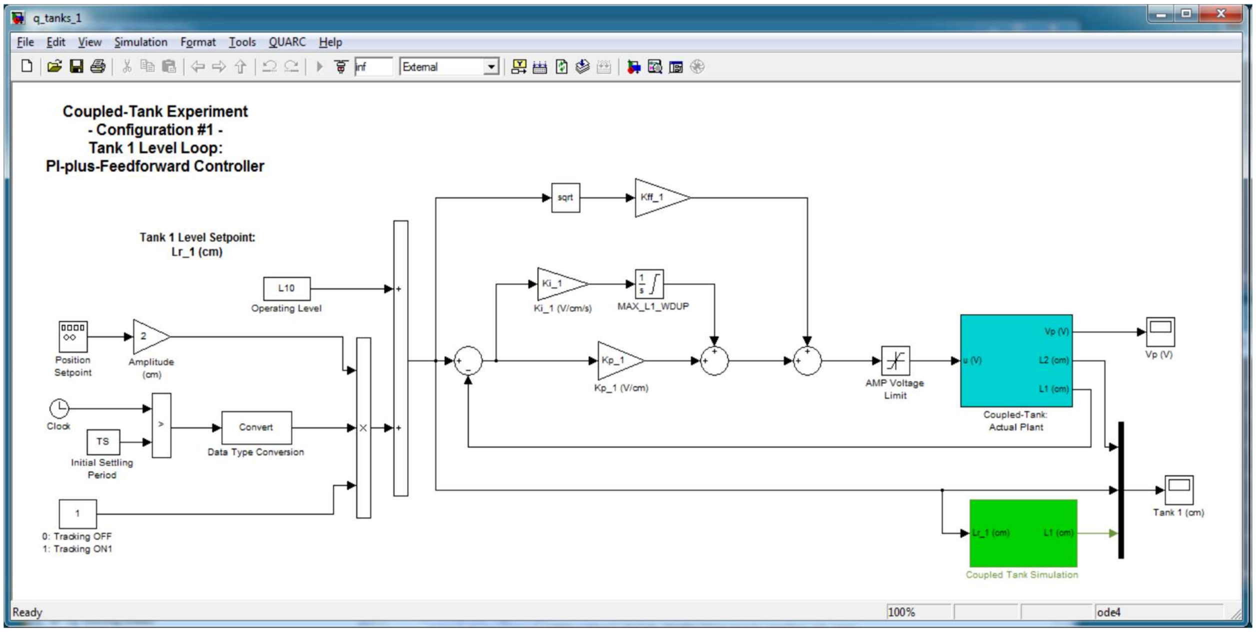


Figure 7.9: Simulink model used to run tank 1 level control on Coupled Tanks system.

Follow this procedure:

1. Enter the proportional, integral, and feed forward control gains obtained, in Matlab Kp_1, Ki_1 and Kff_1.
2. To generate a step reference, go to the Signal Generator block and set it to the following:
    - Signal type = square
    - Amplitude = 1
    - Frequency = 0.06 Hz
3. Set the Amplitude (cm) gain block to 1 to generate a square wave goes between 1 cm.
4. Open the pump voltage Vp (V) and tank 1 level response Tank 1(cm) scopes.
5. By default, there should be anti-windup on the Integrator block (i.e., just use the default Integrator block).
6. Create a subsystem of the Actual plant (with HIL read and write analog as stated in section IX, X)
7. In the Simulink diagram, go to QUARC | Build.
8. Click on QUARC | Start to run the controller.

    The pump should start running and filling up tank 1 to its operating level, L_{10}. After a settling delay, the water level in tank 1 should begin tracking the $\pm$ 1 cm square wave set point (about operating level L_{10}).
9. Generate a Matlab figure showing the *Implemented Tank 1 Control* response and the input pump voltage.
10. Measure the steady-state error, the percent overshoot and the peak time of the response.

    Hint: Use the Matlab *ginput* command to take measurements off the figure

XI. Coupled Tank Model Parameters

Below table, lists and characterizes the main parameters associated with the two-tank specialty plant. Some of these parameters can be used for mathematical modelling of the Coupled-Tank system as well as to obtain the water level's Equation Of Motion (EOM).

Symbol	Description	Value	Unit
K_P	Pump Flow Constant	3.3	$cm^3/s/V$
V_{Pmax}	Pump Maximum Continuous Voltage	12	V
V_{Ppeak}	Pump Peak Voltage	22	V
D_{Out1}	Out 1 Orifice Diameter	0.635	cm
D_{Out2}	Out 2 Orifice Diameter	0.47625	cm
L_{1max}	Tank 1 Height (i.e. Water Level Range)	30	cm
D_{t1}	Tank 1 Inside Diameter	4.445	cm
K_{L1}	Tank 1 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
K_{L2}	Tank 2 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
V_{bias}	Tank 1 and Tank 2 Pressure Sensor Power Bias	+/-12	V
P_{range}	Tank 1 and Tank 2 Sensor Pressure Range	0 - 6.89	kPa
D_{So}	Small Outflow Orifice Diameter	0.31750	cm
D_{Mo}	Medium Outflow Orifice Diameter	0.47625	cm
D_{Lo}	Large Outflow Orifice Diameter	0.55563	cm
g	Gravitational Constant on Earth	981	cm/s^2

XII. Experimental Procedure

1.
   a. Derive the Equation Of Motion (EOM) characterizing the dynamics of tank 1. Is the tank 1 system's EOM linear?

      Hint: The outflow rate from tank 1, $Fo1$, can be expressed by: $Fo1 = Ao1vo1$

   b. Find the nominal pump voltage Vp0 at system's static equilibrium. By definition, static equilibrium at a nominal operating point (Vp0;L10) is characterized by the water in tank 1 being at a constant position level L10 due to the constant inflow rate generated by Vp0. (Assume L10=15cm).

   c. Linearize tank 1 water level's EOM found in Problem 1 a. about the quiescent operating point ($Vp0;L10$).

   d. Express the open-loop transfer function DC gain Kdc1 and τ_1 as functions of $L10$ and the system parameters. What is the order and type of the system? Is it stable? Evaluate according to system's specifications.

2.
   a. Analyze tank 1 water level closed-loop system at the static equilibrium point ($Vp0;\ L10$) and determine and evaluate the voltage feedforward gain, K_{ff_1}.

   b. Using tank 1 voltage-to-level transfer function $G1(s)$ and the control scheme block diagram illustrated in Figure 7.2, derive the normalized characteristic equation of the water level closed-loop system.

Hint: The feedforward gain Kff_1 does not influence the system characteristic equation. Therefore, the

Feedforward action can be neglected for the purpose of determining the denominator of the closedloop transfer function. Block diagram reduction can be carried out)

c. By identifying the controller gains *Kp*_1 and *Ki*_1, fit the obtained characteristic equation to the second order standard form.

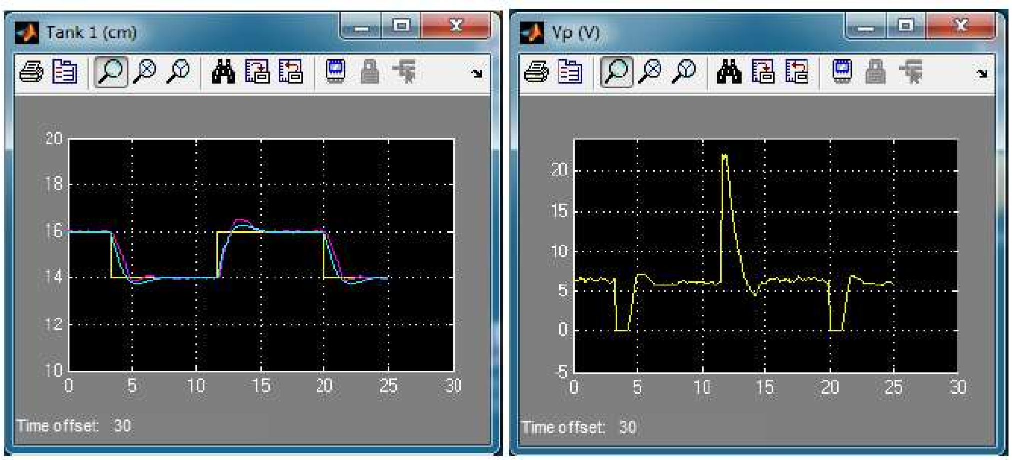
d. Determine the numerical values for *Kp*_1 and *Ki*_1 in order for the tank 1 system to meet the closed loop desired specifications, as previously stated.

3.

a. Perform Level control simulation as described in Section XI

b. Assess the actual performance of the level response and compare it to the design requirements. Measure your response actual percent overshoot and settling time. Are the design specifications satisfied? Explain. If your level response does not meet the desired design specifications, review your PI-plus-Feedforward gain calculations and/or alter the closed-loop pole locations until they do.

4. Perform Tank 1 Level Control Implementation, Complete the steps listed in Section X of this manual. Use below figure for reference.



(a) Tank 1 Level (b) Pump Voltage

Fig. Measured closed-loop tank 1 control response