EE 216 - Experiment 2
Continuous-Time Signal and System
Models and Characteristics

Objectives:

To learn to use MATLAB to compute and plot data and plot continuous-time signal models. Also, to develop computer solutions for system equations and observe system characteristics.

Theory:

A continuous-time signal is defined for every point in time. This means that there are an infinite number of points in any time interval, regardless of how small the interval is. It also means that the signal is defined for $t = -\infty$ to $t = +\infty$.

Only a finite number of values can be used to represent a continuous-time signal in a digital computer. These values are samples of the signal taken over a finite-length segment of the signal at times equal to integer multiples of a finite sample spacing T. That is, signal samples are taken at $t = nT$ where n is an integer. This signal sampling is illustrated in Figure 2.1. The spacing must be small enough so that the signal is well-represented and smooth plots can be made. The finite segment length must be chosen to adequately portray the signal characteristics.

An impulse cannot be represented by samples since it has finite area at one point in time and thus has infinite height at that point. To be able to work with impulses in computer solutions, we use an approximation to the impulse. To generate this approximation, we recall that an intuitive description for $A\delta(t-nT)$ is the limit as $T \rightarrow 0$ of the continuous-time rectangular pulse signal shown in Figure 2.1.

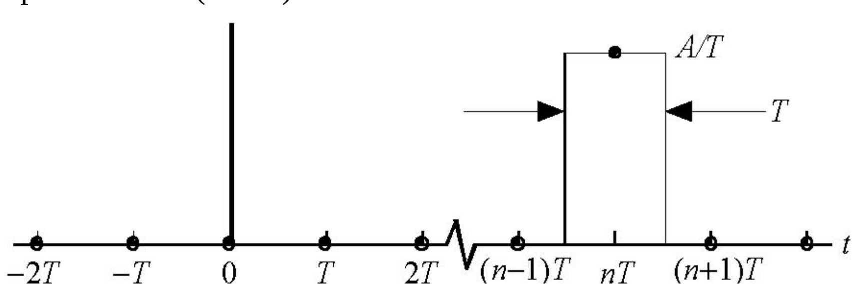

Figure 2.1 Rectangular Pulse of Area A That Approaches the Impulse $A\delta(t-nT)$ as $T \rightarrow 0$

Sample values of this signal, as shown by the dots in Figure 2.1, are all zero except for a sample value of A/T at $t = nT$. Thus, for computation purposes, we can approximate the impulse $A\delta(t-nT)$ by a single nonzero sample of value A/T at $t = nT$. The approximation becomes better as T becomes smaller.

A continuous-time system is characterized by an integro-differential equation. For example, a first-order system is characterized by the first-order differential equation

$$a\frac{dy}{dt}+by(t)=cx(t)\,dy \qquad \textit{(2.1)}$$

where $x(t)$ and $y(t)$ are the system input and output signals, respectively.

Symbolic differential equation solvers exist for digital computers. Such an equation solver gives $y(t)$ as a function of t when $x(t)$ and an initial condition are supplied for eq. (2.1). The equation solver is not always convenient to use and cannot obtain results for all differential equations and all input signals. We will not use it in this laboratory.

Even though we cannot find the exact system output signal as a function of time with the digital computer, we can compute and plot approximate values for its samples. We do this by using an approximate derivative. If the signal sample spacing is T, then we use the approximation

$$\left.\frac{dw}{dt}\right|_{t=nT} \doteq \left\{w(nT)-w((n-1)T)\right\}/T \qquad (2.2)$$

for the derivative at $t = nT$. Therefore, to compute approximate sample values for $y(t)$ in eq. (2.1) at $t = nT$, where $n > n_1$, given samples of the input signal at $t = nT$ and the initial condition value $y(n_1T)$ at $t = t_1 = n_1T$ we first write the equation

$$\frac{a}{T}\left[y(nT)-y((n-1)T)\right]+by\left[nT\right]=c\left[nT\right] \qquad (2.3)$$

This equation can be rewritten as

$$y\left[nt\right]=\frac{1}{a+bT}\left[ay\left[(n-1)T\right]+cTx\left[nT\right]\right] \qquad (2.4)$$

and solved recursively for $y(nT)$ at $n = n_1 + 1$, $n_1 + 2$, etc. The accuracy of the approximate values computed for $y(nT)$ improves as we decrease T.

Example 2.1

Find the recursive approximate solution for samples of $y(t)$ in eq. (2.1) when $t \geq 0$, $a = 1$, $b = c = 2$, $x(t)= 1/(1+t)$, $y(0)=4$, and the derivative approximation is used with $T = 0.01$ [See eq. (2.4)].

Solution:

$$n=1 \quad \mathrm{y(0.01)} = \left(\mathrm{y}(0)+002\mathrm{x}(00.1)\right)/1.02$$
$$= \{4+0.02/1.01/1.02=3.941\}$$

$$n=2 \quad y(0.02) = \left(y(0.01)+0.02x(0.02)\right)/1.02$$
$$= \{3.941+0.02/1.02\}1.02=3.883$$

$$n=3 \quad y(0.03) = \{y(0.02)+0.02x(0.03)\}/1.02$$
$$= \{3.883+0.02/1.03\}/1.02=3.826$$

etc.

We can use an approximate integral rather than an approximate derivative to solve eq. (2.1) for the output signal samples *y*(*nT*). To do so, we first integrate eq. (2.1) from $t=(n-1)T \ \ to \ t=nT$ to give

$$y(nT)=\frac{1}{a}\int_{(n-1)T}^{nT}\left(cx(\tau)-by(\tau)d\tau+y((n-1)T)\right) \tag{2.5}$$

There are a number of different integral approximations. Here we use the trapezoidal rule

$$\int_{(n-1)T}^{nT} w(\tau)d\tau \doteq \frac{T}{2}\left[w(nT)+w((n-1)T)\right] \tag{2.6}$$

Thus,

$$y(nT)\doteq\frac{T}{2a}cx(nT)-by(nT)+cx((n-1)T)-by((n-1)T)+y((n-1)T) \tag{2.7}$$

which can be written as

$$y(nT)\doteq\frac{cT}{2a+bT}\left[x(nT)+x((n-1)T)\right]+\frac{2a-bT}{2a+bT}y((n-1)T) \tag{2.8}$$

and solved recursively for $y(nT)$ at $n=n_1+1, n_2+2,$ etc. Again, the accuracy of the approximate values computed for $y(nT)$ improves as we decrease T.

Example 2.2

Repeat Example 2.1 using the integral approximation [see eq. (2.8)]

Solution:

$$n=1 \quad y(0.01) = \left[0.02\left[x(0.01)+x(0)\right]+1.98y(0)\right]/2.02$$
$$= \left(0.02\left[1/1.01+1\right]+1.98(4)\right)/2.02 = 3.941$$
$$n=2 \quad y(0.02) = \left[0.02\left[x(0.02)+x(0.01)\right]+1.98y(0.01)\right]/2.02$$
$$= \left(0.02\left[1/1.02+1/1.01\right]+1.98(3.941)\right)/2.02 = 3.882$$
$$n=3 \quad y(0.03) = \left[0.02\left[x(0.03)+x(0.02)\right]+1.98(0.02)\right]/2.02$$

Etc.

Note that the results are slightly different in Examples 2.1 and 2.2. This is because both solutions are approximate and T is not very small.

Preliminary:

1. Use the derivative approximation to find the approximate equation corresponding to the second-order differential equation

$$0.3\frac{d^2y(t)}{dt^2}+0.4\frac{dy(t)}{dt}+0.8y(t)=0.9x(t)+0.2\frac{dx(t)}{dt}$$

so that you can use it to solve for sample values of $y(t)$. **Hint:** The second order derivative corresponds to applying the derivative approximation twice.

2. Find the derivative and integral approximations for the equation **x(t)** in Part 3.

Laboratory Procedure:

NOTE: Read the calculations section prior to beginning the laboratory work.

1. The two currents flowing into a circuit node are $i_1(t) = 2.4\cos(15\pi t - 0.8)$ and $i_2(t) = 4.2\cos(15\pi t - 1.9)$. Plot the currents and the current that exits the node on the same set of axes for $-0.1 \le t \le 0.1$.

2. Plot the signal

$$x(t) = 2u(t+2.5)+2r(t+2)-10u(t+1)-r(t)-r(t-1)$$
$$+u(t-1)+0.5r(t-2.5)-0.5r(t-6.5)$$

Use the step and ramp functions **us(t)** and **ur(t)** described in Section 2.1.1 of the tutorial to compute data for the plot.

3. The RC low-pass filter shown in Figure 2.3

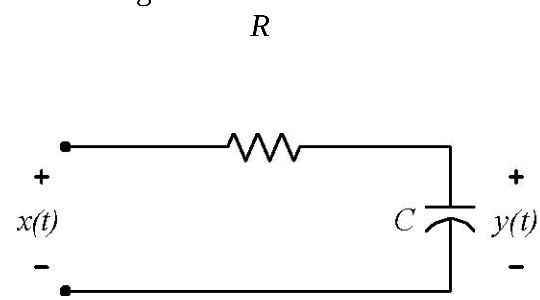

Figure 2.3 RC Lowpass Filter

is characterized by the system equation

$$x(t) = RC\frac{dy(t)}{dt} + y(t)$$

Plot the input and output signals for $0.6 \le t \le 1.6$ when $x(t) = t\left[u\left(t-0.8\right) - u\left(t=1.1\right)\right]$, $R = 2k\Omega,\ C = 10\mu$F, and the capacitor current is initially zero. Use the derivative approximation and try several values of *T*. Approximately, what is the largest value of *T* that appears to give good results?

4. Repeat Part 3 using the integral approximation. Compare the results with Part 3.

5. The impulse response of the RC Lowpass filter shown in Figure 2.3 is $h(t) = \frac{1}{RC}e^{-\frac{t}{RC}}u(t)$. Plot the approximate impulse response found by using MATLAB to solve the differential equation corresponding to the filter when $R = 2k\Omega$ and $C = 10\mu\text{F}$. Plot the actual impulse response on the same set of axes that encompass the interval $-0.1 \le t \le 0.2$. Vary the value of *T* to illustrate the effect of sample spacing on the approximation accuracy.

6. Plot *x*(*t*) and *y*(*t*) corresponding to the differential equation in Part 1 of the preliminary for $0 \le t \le 10$ when $y(0) = 2,\ y(T) = 2 - 0.35T$, and the signal $x(t)$ is the piecewise-defined signal

$$x(t) = \begin{cases} -3t+6 & 1 \le t < 2 \\ -3t+9 & 2 \le t < 3 \\ -3t+12 & 3 \le \text{t} < 4 \\ 0 & \text{elsewhere} \end{cases}$$

(Note: this signal can be defined with two ramps and three steps.) Two initial conditions are required for $y(t)$ since the differential equation is second-order. You will therefore begin your solution with $y(2T)$. The initial conditions correspond to an initial value of 2 and an initial slope of -0.35.

Calculations:

1. For Part 1 of the Laboratory Procedure, find the mathematical expression for the current that exits the node. Calculate the time delay of each current signal with respect to a reference cosine. Visually inspect the curves plotted and comment on correspondence of computed and observed current time delays and exit current magnitude.