Introduction

Since its invention by Nikola Tesla in the 1800s, the induction motor has remained the most popular type of motor for industry applications. The primary advantage of the induction motor is its straightforward rotor construction leading to low cost, ruggedness, and low-maintenance requirements.

Induction motor model

As with other electrical machines, an equivalent electric circuit can be developed for the purpose of gaining insight into the motor's operation. The per-phase equivalent circuit of an induction motor is shown in Figure 1. The circuit impedances are defined (in ohms) by the following:

- $R_1$ stator resistance
- $X_1$ stator leakage reactance
- $X_m$ magnetizing reactance
- $R_c$ machine core loss resistor
- $X_2$ rotor leakage reactance
- $R_2$ rotor resistance

The terminals $a$ and $n$ represent the $a$-phase and neutral points of a wye-connected induction motor. Similar equivalent circuits exist for the $b$- and $c$- phases with all quantities having the same values but lagging the $a$-phase by $120^\circ$ and $240^\circ$ respectively.

Figure 1. Per-phase equivalent circuit of an induction motor.
Note that the core loss resistor element is added to account for hysteresis and eddy currents. Since these losses are a function of frequency, this term is only approximate and is sometimes not included in the model. The circuit electrical quantities are:

\[
\begin{align*}
\hat{V}_1 & \quad \text{motor phase voltage (V)} \\
\hat{I}_1 & \quad \text{stator current into motor phase (A)} \\
\hat{I}_2 & \quad \text{rotor current (A)} \\
\hat{V}_m & \quad \text{voltage across magnetizing circuit (V)} \\
\hat{I}_m & \quad \text{magnetizing current (A)} \\
s & \quad \text{per unit slip}
\end{align*}
\]

Herein, the hat symbol ^ represents a phasor. The magnitude of the phasor is denoted by the variable of the same name (i.e. \( I_1 \) is the magnitude of \( \hat{I}_1 \)). The phase voltage \( \hat{V}_1 \) is used as the phasor reference with an angle of zero. The per unit slip \( s \) is defined by

\[
s = \frac{\omega_r - \omega_m}{\omega_s} \quad (1)
\]

In (1), \( \omega_m \) is the rotor mechanical speed and \( \omega_s \) is the synchronous speed of the stator poles which is a function of the electrical frequency of the excitation source \( \hat{V}_1 \) and the number of stator poles. For a machine operating at a frequency \( f \), the synchronous speed can be calculated (in rad/sec) from

\[
\omega_s = \left( \frac{2 \text{ poles}}{\text{poles}} \right)(2\pi f) \quad (2)
\]

To compute the per-phase input impedance of the induction motor, it is convenient to define the stator, magnetizing, rotor, and "field" impedances respectively as

\[
\begin{align*}
Z_1 & = R_1 + j X_1 \\
Z_m & = R_v \parallel j X_m \\
Z_2 & = \frac{R_2}{s} + j X_2 \\
Z_t & = Z_m \parallel Z_2
\end{align*}
\]

The input impedance is then given by

\[
Z_{in} = Z_1 + Z_t = Z \angle \theta \quad (7)
\]
The angle of the input impedance is the negative of the angle of $\hat{I}_1$ since the phase voltage is used as the reference. The cosine of this angle is the power factor

$$p f = \cos(\theta) \quad (8)$$

Three simple laboratory tests can be used to determine the circuit model impedance parameters. These tests are described below.

**Measuring equivalent circuit model parameters**

The simplest element to determine is the stator resistance $R_1$ which can be measured with an ohmmeter. However, for large machines it is usually measured with a DC source so that it can be determined at the rated value of current; thereby including the increase in resistance with temperature rise. From Figure 1, it can be seen that if a DC source is used the input impedance contains only $R_1$ since the inductors will act as short circuits. Figure 2 shows the per-phase equivalent circuit that exists when a DC voltage is applied to the A-phase. The applied DC voltage and resulting DC current are measured and the stator resistance is calculated from

$$R_1 = \frac{V_{DC}}{I_{DC}} \quad (9)$$

![Figure 2. Equivalent circuit for the DC resistance test.](image)

After determining the stator resistance, a blocked rotor test is performed on the induction motor. This involves physically blocking the rotor and applying a low voltage so that rated current flows in the stator windings. Under these conditions, the mechanical speed $\omega_m$ is zero and therefore the slip is unity. Including all parameters, we end up with complicated relationships between the machine parameters and the measurements. However, under the condition that $X_2 \ll X_m$ (which is reasonable in most practical machines), we can simplify the reactance expression somewhat.

By measuring the voltage, current, and input power, the blocked rotor impedance can be calculated by
Induction Motor Parameter Measurement

\[ Z_{bl} = \frac{V_{1,bl}}{I_{1,bl}} \angle \tan^{-1}\left( \frac{Q_{bl}}{P_{bl}} \right) \]  \hspace{1cm} (10)

The real and imaginary parts of this impedance determine the resistances and reactances of the machine respectively as

\[ \text{Re}\{Z_{bl}\} = R_1 + \left( \frac{X_m}{X_2 + X_m} \right)^2 R_2 \]  \hspace{1cm} (11)

\[ \text{Im}\{Z_{bl}\} = \frac{X_1 X_2 + X_1 X_m + X_2 X_m}{X_2 + X_m} \]  \hspace{1cm} (12)

Under the assumption that \( X_2 \ll X_m \),

\[ \text{Im}\{Z_{bl}\} = X_1 + X_2 \]  \hspace{1cm} (13)

In order to determine the stator and rotor leakage reactances, knowledge of the machine construction is necessary. Table I displays the necessary relationship between \( X_1 \) and \( X_2 \) for the most common types of industrial induction motors. These are classified by the National Electrical Manufacturers Association (NEMA).

<table>
<thead>
<tr>
<th>Table I. NEMA class motor data.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NEMA class A</td>
</tr>
<tr>
<td>NEMA class B</td>
</tr>
<tr>
<td>NEMA class C</td>
</tr>
<tr>
<td>NEMA class D</td>
</tr>
</tbody>
</table>

NEMA classes are based on the motor performance which is a function of motor geometry. Performance characteristics include speed regulation, stall torque, and starting current. The most common type of induction motor is the NEMA class B since its performance is a good compromise of the performance characteristics. Table I combined with (13) is sufficient to find the leakage reactances. More information is needed, though, to find \( R_2 \), \( R_c \), and \( X_m \).

The remaining parameters can be determined from a no-load test. If the motor is operating at rated voltage and without a load, the mechanical speed is close to synchronous and the slip is zero (neglecting mechanical friction load). With a slip of zero, the rotor impedance goes to infinity and the equivalent electric circuit is that shown in Figure 3. As with the blocked rotor test, the voltage, current, and input power are measured and the no-load input impedance is calculated from

\[ Z_{nl} = \frac{V_{1,nl}}{I_{1,nl}} \angle \tan^{-1}\left( \frac{Q_{nl}}{P_{nl}} \right) \]  \hspace{1cm} (14)
Most commonly, $R_c$ is relocated to the input terminals. As long as it is large (compared to $R_1$ and $X_1$), this approximation is sufficient. Then

$$R_c = \text{Re}\{Z_{nl}\} \quad (15)$$

$$X_1 + X_m = \text{Im}\{Z_{nl}\} \quad (16)$$

Combining (16) with the blocked-rotor results, we can find $X_m$. Now we have sufficient information (including knowledge of $R_1$ from the dc test) to use (11) to find $R_2$.

Figure 3. Equivalent circuit for no-load test.
Laboratory Software

Screenshots of the laboratory software are shown in Figure 4. The Test Type allows selection between the three tests. For the DC resistance test (shown in the top of Figure 4), a bar is included for commanding a dc current into the motor. The voltage, current, and power can be logged. When the Test Type is set to "No Load" the meters change from DC to AC and voltage and current waveform plots appear. Note that this program already divides the line-to-line voltage by $\sqrt{3}$ so that line-to-neutral voltages are displayed and logged. The blocked rotor test screen is similar to the no-load test screen with the exception that the data is logged with a different label.

![Figure 4. Induction motor parameter measurement program screen-shot.](image)
Laboratory Work

Laboratory Induction Motors

The laboratory induction motors are standard NEMA class B industrial machines with ratings shown in the table below.

<table>
<thead>
<tr>
<th>poles = 4</th>
<th>$f = 60$Hz</th>
<th>$n = 1725$RPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{LL} = 208$V</td>
<td>$I_L = 3.1$A</td>
<td>$P_{shaft} = 1$hp</td>
</tr>
</tbody>
</table>

DC Resistance Test

Connect the induction motor to the Magna-Power DC power supply (labeled “Sorenson” here for historical reasons) through the meter panel as shown in Figure 5. Select "DC Test" from the tab in the computer program and click "Zero DC Offset".

Energize the Magna-Power power supply. On the computer screen, increase the commanded current to 3 A. According to the machine nameplate, this is close to the rated current. Log the data by clicking Add.

Reduce the commanded current to zero and switch off the Magna-Power power supply.

Note that this resistance was measured from line-to-line. The per-phase stator resistance will be half of the value calculated from this measurement.

Blocked Rotor Test

Connect the induction motor to the source panel as shown in Figure 6. Select "Blocked Rotor" from the tab in the computer program.

Make sure the source voltage is set to zero and then switch on the source panel circuit breaker. Hold the motor shaft and increase the source voltage until rated current is reached. Note that this will be a relatively low voltage (with the source knob set to about 20%). Log the data by clicking Add.

Decrease the source panel voltage to zero and switch off the source panel.

No-Load Test

For the no-load test, the same electrical connections as the blocked rotor test (Figure 6) will be used. Change the test type in the software to "No Load".

Switch on the source panel circuit breaker and increase the voltage to 100%. At this point, the motor has rated voltage applied and is operating near synchronous speed. Log the data by clicking Add.

Decrease the source voltage to zero and switch the circuit breaker off.
Figure 5. Laboratory setup for the dc resistance test.
Figure 6. Laboratory setup for the blocked rotor and no-load tests.
Calculations and Questions

1. Based on the machine ratings, compute the slip at rated speed. Also compute the torque (in Newton-meters), input power (in Watts), and power factor at rated speed.

2. Using the DC resistance test measurements, compute the stator resistance.

3. Using the blocked rotor test, compute the rotor resistance as well as the stator and rotor leakage reactances. Use the relationship in Table I to determine the distribution of leakage reactances.

4. From the no-load test data, determine the magnetizing reactance and the core loss resistance.

5. Compile the complete set of machine parameters in a table along with the machine ratings. List the ratings in metric units (e.g. watts and rad/s).