Single-Phase Transformers

Introduction

Figure 1 shows the transformer schematic symbol and the corresponding commonly used Steinmetz model is shown in Figure 2. Therein, the model voltage and current phasors are defined as:

- $\hat{V}_1$: Primary side voltage (V)
- $\hat{I}_1$: Primary side current (A)
- $\hat{V}_2'$: Referred secondary side voltage (V)
- $\hat{I}_2'$: Referred secondary side current (A)
- $\hat{I}_\phi$: Magnetizing current (A)

The model parameters are:

- $R_1$: Primary coil resistance (Ω)
- $X_{li}$: Primary coil leakage reactance (H)
- $R_c$: Core loss resistance (Ω)
- $X_m$: Core magnetizing reactance (Ω)
- $R_2'$: Referred secondary coil resistance (Ω)
- $X_{l2}'$: Referred secondary coil leakage reactance (H)
- $\frac{N_1}{N_2}$: Transformer turns ratio

![Figure 1. Transformer symbol.](image1)

![Figure 2. Steinmetz model of a transformer.](image2)
Note that this Steinmetz model utilizes the secondary quantities referred to the primary which are related to the physical quantities by

\[ \hat{V}_2' = \left( \frac{N_2}{N_1} \right) \hat{V}_2 \]  

\[ \hat{I}_2' = \left( \frac{N_2}{N_1} \right) \hat{I}_2 \]  

\[ R_2' = \left( \frac{N_1}{N_2} \right)^2 R_2 \]  

\[ X_{12}' = \left( \frac{N_1}{N_2} \right)^2 X_{12} \]  

Furthermore, this Steinmetz model has be augmented with a core loss term \( R_c \) in parallel with \( X_m \) that approximately accounts for the hysteresis and eddy current losses in the core.

**Measuring Winding Resistance**

The resistance parameters \( R_1 \) and \( R_2 \) can be found by applying a dc voltage to the respective windings. For example, if a dc voltage is applied to the primary winding, the inductors become short circuits and the Steinmetz model predicts the equivalent circuit shown in Figure 3. The primary resistance can then be determined by

\[ R_1 = \frac{V_{dc}}{I_{dc}} \]  

The same procedure can be used to find the secondary resistance \( R_2 \). Note that the turns ratio is needed to refer this resistance to the primary according to (3). The turns ratio is usually given by the transformer manufacturer as the ratio of the rated primary voltage to the rated secondary voltage.
Measuring Leakage Reactances

If the secondary of the transformer is short-circuited, the magnetizing current $\hat{I}_p$ is negligible and the equivalent circuit can be approximated as shown in Figure 4. In this case, the magnetizing impedance is assumed to be much larger than the secondary resistance and secondary leakage reactance. This is typically a good assumption for practical transformer designs. From the voltage, current, and power measurements, the magnitude and angle of the short-circuit impedance can be determined as

$$VA_{sc} = \left| \frac{\hat{V}_{sc}}{\hat{i}_{sc}} \right|$$  \hspace{1cm} (6)

$$|Z_{sc}| = \left| \frac{\hat{V}_{sc}}{\hat{i}_{sc}} \right|$$  \hspace{1cm} (7)

$$\angle Z_{sc} = \cos^{-1}\left( \frac{P_{sc}}{VA_{sc}} \right)$$  \hspace{1cm} (8)

From the imaginary part of this impedance, the sum of the primary and secondary leakage reactances can be found. In particular,

$$X_{l1} + X_{l2}' = \text{Im}\{Z_{sc}\}$$  \hspace{1cm} (9)

The individual leakage reactances can be determined if it is assumed that

$$X_{l1} = X_{l2}'$$  \hspace{1cm} (10)

Although it is possible to perform other tests on the transformer to determine a more exact relationship between $X_{l1}$ and $X_{l2}'$, (10) is a good approximation.

Figure 4. Approximated Steinmetz model for short-circuit test.
Measuring Magnetizing Reactance and Core Losses

The next test involves open-circuiting the secondary winding and applying rated voltage to the primary winding. Under this condition, the Steinmetz equivalent circuit (including the core loss term) reduces to that shown in Figure 5. From measured voltage, current, and power, the open-circuit impedance can be found using

\[ V_{oc} = \left| \hat{V}_{oc} \right| \left| \hat{i}_{oc} \right| \quad (11) \]

\[ |Z_{oc}| = \left| \frac{\hat{V}_{oc}}{\hat{i}_{oc}} \right| \quad (12) \]

\[ \angle Z_{oc} = \cos^{-1}\left( \frac{P_{oc}}{S_{oc}} \right) \quad (13) \]

![Figure 5. Steinmetz model for open-circuit test.](image)

Using the measured open-circuit impedance as well as \( R_1 \) and \( X_{li} \) values from the previous tests, the core admittance can be determined by

\[ Y_c = \frac{1}{Z_{oc} - (R_1 + jX_{li})} \quad (14) \]

From this admittance, the magnetizing reactance and core loss resistance can be found from

\[ X_m = \frac{-1}{\text{Im}\{Y_c\}} \quad (15) \]

\[ R_c = \frac{1}{\text{Re}\{Y_c\}} \quad (16) \]
Laboratory Software

Figure 6 shows a screen-shot of the program used for the single-phase transformer experiments. This program monitors the first two channels of the meter box and displays their waveforms and measurements. Data can be logged by selecting the type of test and clicking Add. The type of test can be selected from the drop-down Test menu. When the DC Test is selected (as shown in Figure 6), channel 1 will read dc values. Due to analog offsets, the dc voltage may have a small value before the measurement is taken (when the dc voltage and current are zero). In this case, the Zero DC Offset button can be clicked to zero the meters. When ac tests, such as Short-Circuit and Open-Circuit, are selected, the meter on channel 1 reads rms values.

![Single-Phase Transformer](image)

Figure 6. Laboratory software interface for the single-phase transformer experiment.
Laboratory Transformers

Figure 7 shows the laboratory transformer and a corresponding electrical connection diagram. The transformer has multiple primary and secondary windings which may be connected in series or parallel for different voltage ratings. Internally, the primary windings have been connected in series and the secondary has been connected in parallel yielding the ratings show in the table below.

<table>
<thead>
<tr>
<th>Laboratory transformer ratings.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{V}_1 ) = 240 V</td>
</tr>
<tr>
<td>( \hat{I}_1 ) = 4.17 A</td>
</tr>
</tbody>
</table>

From the voltage ratings, it can be seen that the turns ratio is \( \frac{N_1}{N_2} = 2 \). The connection diagram also shows the primary taps which are accessible through the connectors.

As can be seen, primary taps at 120-V and 208-V are available using terminals H2-H3 and H4 respectively. In low-power single-phase residential applications, 120-V is a common voltage level. This is typically obtained by splitting a 240-V winding. Alternatively, 120-V can be obtained from the line-to-neutral voltage of a 208-V three-phase system. For these reasons, 120-V and 208-V tap settings are commonly available on many 240-V transformers.
Laboratory Work

DC Resistance Test

Connect the transformer as shown in Figure 8 using two-conductor black (B) and white (W) cable. For this test, resistance from the load box will be placed in series with the source to limit the current. Note that all three phases of the load box are connected in parallel using the additional jumper wires on the load box connector. The load box should be set to use two of the resistors in each phase as shown in Figure 9. Turn on the load box fans. For this test, click the "DC Test" tab in the software display. This test must be selected for the meter to read current from a dc source! Switch on the source panel circuit breaker and increase the voltage until the dc current is 4.17-A. Add the data to the log (you can also type "Primary" next to DC Test). Decrease the voltage and switch off the circuit breaker.

Move the transformer connections from the H1 and H5 terminals to the X1-X3 and X2-X4 terminals. Repeat the dc resistance test for the secondary side. For this test, the voltage should be increased until the dc current is 8.33-A.

Figure 8. Laboratory connections for the dc resistance test.
Figure 9. Load box settings for the dc resistance test.
**Short-Circuit Test**

Using two phases of the AC source panel and a resistance from the load panel, connect the transformer as shown in Figure 10. Note that this is similar to the dc resistance test, except for the source connections and the short-circuit wire on the transformer secondary side. The load box settings are also the same as before. Select "Short-Circuit" from the Test type in the software. Increase the voltage applied to the primary until the rms primary current reaches 4.17-A. Log the voltage and current values by clicking Add. Reduce the voltage to zero and switch off the source circuit breaker.

![Figure 10. Laboratory connections for the short-circuit test.](image)
Open-Circuit Test

The lab set-up for the open-circuit test is similar to that of the short-circuit test. To set-up for the open-circuit test, remove the short-circuit wire on the transformer secondary. Switch on the source circuit breaker and increase the voltage to 100%. This is near rated voltage for the transformer. Log the data values and note the primary current waveform. This is the transformer exciting current and has a non-linear shape due to transformer saturation. Print the waveform from the screen. Reduce the voltage and switch off the source circuit breaker.

Transformer Load Test

Connect the transformer to the three-phase source and load as shown in Figure 11. For the first test, the load box settings from before (Figure 9) can be used. Switch on the source circuit breaker and increase the voltage to 100%. Log the primary and secondary data. Reduce the voltage to zero and switch off the source circuit breaker.

Repeat the load test for the R-L and R-C load cases using the load box settings shown in Figures 12 and 13 respectively.

Click Save to save the voltage, current, and power measurements from the transformer tests.
Figure 11. Laboratory connections for the load test.
Figure 12. Load box settings for the R-L load.

Figure 13. Load box settings for the R-C load.
Calculations

1. From the dc resistance test, short-circuit test, and open-circuit test, calculate the Steinmetz model parameters $R_1$, $X_{f1}$, $R_2$, $X_{f2}$, $R_c$, and $X_m$.

2. Using the transformer turns ratio, calculate the values of the referred load impedances for each of the three load tests (R, R-L, and R-C). The individual values of R, L, and C can be obtained from the Single- and Three-Phase Power Measurements experiment. Note the series and parallel combinations used in this experiment to determine the actual values.

3. Using the transformer model and referred loads, compute the voltages and currents that correspond to each of the three load tests. Use the primary voltage from the load tests and compute the primary current and power as well as the secondary voltage, current, and power. Compare these values to the measured values in a table along with percent errors.