EE 216 - Experiment 9
Ideal Filters

Objectives:

To illustrate the characteristics of ideal filters and their applications to signal analysis.

Theory:

Many times a received signal contains the desired message signal and additive interference or noise. We can reduce the interference if it has components at frequencies outside the frequency band that contains the significant signal components. This reduction is achieved by using a system that has considerably less gain outside the signal frequency band than inside the band. The ideal situation would be if the filter had zero gain outside the signal frequency band and constant gain inside the signal frequency band. If the filter phase response is also a straight line through the origin, then all signal components receive the same gain and time delay in passing through the filter. We say that the signal passes through the filter with no distortion since its only change is in amplitude and location on the time axis. That is, its shape remains the same.

Filter types of interest to us include the low-pass filter (LPF), high-pass filter (HPF), band-pass filter (BPF), and band-rejection filter (BRF). An ideal LPF passes all signal components having frequencies less than B Hz with no distortion and completely attenuates all signal components having frequencies greater than B Hz. Similar definitions can be given for the HPF, BPF, and BRF. Thus, the filter frequency responses for filters of bandwidth B are given by

$$H_L(f) = K\Pi\left(\frac{f}{2B}\right)e^{-j2\pi f t_o} \tag{9.1}$$

$$H_H(f) = K\left[1-\Pi\left(\frac{f}{2B}\right)\right]e^{-j2\pi f t_o} \tag{9.2}$$

$$H_B(f) = K\left\{\Pi\left(\frac{f+f_o}{B}\right)+\Pi\left(\frac{f-f_o}{B}\right)\right\}e^{-j2\pi t_o} \tag{9.3}$$

$$H_R(f) = K\left\{1-\Pi\left(\frac{f+f_o}{B}\right)-\Pi\left(\frac{f-f_o}{B}\right)\right\}e^{-j2\pi f t_o} \tag{9.4}$$

Note that

$$H_H(f) = Ke^{-j2\pi f t_o} - H_L(f) \tag{9.5}$$

and

$$H_R(f) = Ke^{-j2\pi f t_o} - H_B(f) \tag{9.6}$$

Ideal filters are useful in mathematical analysis of the effects of filters on signals. This is because the ideal filter frequency responses are simple functions. They can also be used to analyze easily the contribution of various frequency components in a signal.

Ideal filters can be used in the mathematical model of a system to analyze the system's performance. However, the result is approximate since ideal filters cannot be constructed. The main reason for this is that the ideal filters are all non-causal. That is, their impulse response is nonzero for $t < 0$. Thus, their output at any time depends on inputs at all future times. This characteristic does not affect mathematical analysis but does affect physical systems since a physical filter cannot know what future inputs are unless filtering is performed after the entire signal has been received.

Preliminary:

1. Find the impulse response for a LPF with gain equal to 1.5, bandwidth equal to 1.25 Hz, and time delay equal to t_o
2. Repeat Part 1 for a BPF with gain equal to 1.5, bandwidth equal to 2.5 Hz, center frequency equal to 6, and time delay equal to t_o

Laboratory Procedure:

1. An ideal LPF has gain equal to 1.5, bandwidth equal to 1.25 Hz, and time delay equal to zero.

   a. Compute samples of the frequency response over the interval $-25 \le f \le 25$ using a sample spacing of 0.01 Hz. Plot the amplitude and phase response for $-5 \le f \le 5Hz$.

   b. Use the frequency response samples computed in Part 1a to compute samples for the filter impulse response. Use a maximum impulse response sample spacing of 0.02 s. Plot the impulse response for $-1.5 \le t \le 2.5\ s$ .Also plot the impulse response computed in the Preliminary for comparison purposes.

2. Repeat Part 1 for a BPF with gain equal to 1.5, bandwidth equal to 2.5 Hz, center frequency equal to 6 Hz, and time delay equa l to zero. In Part a, plot the amplitude and phase response for $-10 \le f \le 10Hz$.

3. Samples of the electrocardiograph signal corresponding to three beats of a human heart are contained as variable **hb** in file **hum3hb.mat**. Also contained in this file is the sample spacing *T*. The signal contains noise and hum from the electrocardiograph 60 Hz power supply.

   a. Find the spectrum for the signal. Plot the signal and the amplitude spectrum of the signal. Plot the amplitude spectrum for $-150 \le f \le 150$ Hz Identify the spectrum terms that correspond to the power supply hum.

b. Construct an ideal BRF with zero time delay to remove the power supply hum terms. Each rejection band is to be 10 Hz. wide. Pass the signal spectrum through the BRF and plot the filter output signal and its amplitude spectrum. Comment on the effect of the filter on the signal.

c. Plot the hum signal removed and its amplitude spectrum.

d. In part 3b, you should have noticed that noise still remains on the signal. To remove much of the noise, pass the signal through an ideal LPF with cutoff frequency equal to 55 Hz and time delay equal to zero. Plot the filter output signal and its amplitude spectrum. Has the noise been reduced? Do the significant signal features remain?

e. Plot the noise signal removed and its amplitude spectrum.