EE 216 -Experiment 5
Spectra of Periodic Signals

Objectives:

To find the amplitude and phase spectra of periodic signals by using the Fourier Series.

Theory:

A periodic signal $x(t)$ with period T_o and finite energy in each period can be approximated by the finite-length complex-exponential Fourier series.

$$\hat{x}_N(t) = \sum_{n=-N}^{N} X_n e^{j2\pi n f_o t} \tag{5.1}$$

In this series, the series coefficients are

$$X_n = \frac{1}{T_o}\int_{t_1}^{t_1+T_o} x(t) e^{-j2\pi n T_o} dt \tag{5.2}$$

and $f_o = 1/T_o$ is the fundamental frequency of the signal.

The Fourier series is periodic with period equal to its expansion interval length. Since we have chosen the expansion interval to be equal to the signal period, then, $\hat{x}_N(t)$ approximates $x(t)$ in the same way in each period. The approximation is in the minimum integral square error (minimum energy in the error in each period) sense. As we let the number of terms approach inifity (that is, let $N \to \infty$), the energy in the error approaches zero. If $x(t)$ satisfies the Direichlet conditions then $\hat{x}(t) = \lim_{N\to\infty} \hat{x}_N(t)$ equals $x(t)$ at all points where $x(t)$ is continuous. It equals the value at the middle of discontinuities $x(t)$. Thus we can write

$$x(t) = \hat{x}(t) = \sum_{n=-\infty}^{\infty} X_n e^{j2\pi n f_o t} \tag{5.3}$$

At discontinuities the Gibbs effect occurs (see the EE265 text); however, this does not affect eq. (5.3) since the area under the Gibbs ripple is zero when $N = \infty$.

Since the Fourier series coefficients, X_n, are the complex magnitudes of complex exponentials, then the values of $|X_n|$ and $\angle X_n$ give the amplitude and phase, respectively, of the signal components at frequencies nf_0 where $-\infty \le n \le \infty$. Thus and $|X_n|$ and $\angle X_n$ provide the double-sided amplitude spectrum and phase spectrum, respectively, of $x(t)$.

In most cases, we are interested in real signals. For real signals $\left|X_{-n}\right| = \left|X_n\right|$ and $\angle X_{-n} = -\angle X_n$. Thus, we can combine the positive and negative frequency exponential terms to produce

$$x(t) = X_o + \sum_{n=0}^{\infty} 2\left|X_n\right| \cos\left(2\pi n f_o t + \angle X_n\right) \tag{5.4}$$

From eq. (5.4), we easily find the single-sided amplitude and phase spectra for the real periodic signal $x(t)$.

In the special case when $x(t)$ is real *and* even, the Fourier series coefficients are also real and even. That is, the values of X_n can be either positive or negative but not complex. In this case, we can plot the total spectrum specified by the real numbers X_n rather than the amplitude and phase spectra. This is more convenient.

MATLAB Functions:

MATLAB does not contain built-in functions that will calculate the Fourier Series. Two functions have been created for this laboratory that calculate the Fourier Series Coefficients and the samples of the Fourier Series representation of a signal. A brief description of these functions are provided here. A detailed explanation is provided in the Appendix.

[Xn,f,ang,No,Fo] = ctfsc(t,x)

This function computes the Fourier series coefficients of the signal x(t) for an expansion interval specified by t. It plots the coefficients, if desired (this plot is of the spectrum of a periodic signal if the expansion interval, t, equals one period). If ns signal samples are used then ns coefficients will be processed. This function is set up for -10*ns*dt<t(1)<10*ns*dt.

Input arguments
t : time array
x : signal array
ev: set =1 if signal is real and even, 0 otherwise

Output arguments
Xn: coefficient array
F: frequency array
Ang: angle array for plottying
No: center coefficient Xn(no)
Fo: fundamental frequency

If the signal is real and even, the Fourier series coefficients should also be real and even, so you can take the real part of Xn before plotting.

[xfs,Xnn] = ctfs(t,Xn,No,Fo,N)

This function computes samples of the Fourier series approximation xfs(t) of the signal x(t) over the the time interval t from 2N+1 of the Fourier series coefficients Xn (centered on X(no)=Xo).

Input arguments
t : time array
Xn: Fourier series coefficients generated by ctfsc
No: center value generated by ctfsc
Fo: fundamental frequency generated by ctfsc
N : desired number of terms for positive frequency

Output arguments
Xnn: the 2N=1 Fourier series coefficients that are retained Xfs: Truncated Fourier series based on the 2N=1 Fourier series coefficients retained.

If you wish to plot the truncated Fourier series, use plot **(t,xfs)**.

Laboratory Procedure:

Note: The script developed for Part 1 can be used, with modifications, for Part 2.

1. To show that the Fourier series coefficients corresponding to a periodic signal do give the signal amplitude and phase spectrum, we consider the signal

$$x(t) = 10\cos(24\pi t - 1.2) - 8\cos(28\pi t + 1.0) + 4\sin(36\pi t + 2.5)$$

   You can easily verify that the period of this signal is 0.5 s.

   a. Plot the signal over $-0.75 \le t \le 0.75$, which is three periods.

   b. Sketch by hand the amplitude and phase spectra for the signal.

   c. Use **ctfsc** to compute the Fourier series coefficients and plot their amplitudes. Use a signal sample spacing of dt =0.0005 extending from $t = -0.25$ to $t = 0.25 - 0.0005 = 0.2495$. Compare these amplitude and angle plots with the sketches you made in Part 1b.

   d. Repeat Part 1c for $0 \le t \le 0.4995$. Comment on similarities and differences.

   e. Repeat Part lc for $-0.2 \le t \le 0.1995$ (three-fourths of one period). Comment on similarities and differences.

f. Use **ctfs** to compute the Fourier series corresponding to the Fourier series coefficients in part 1e. Plot this Fourier series. Comment on why this signal appears the way it does.

2. Using the signal given in part 1e, plot the Fourier series corresponding to the non-negligible (greater than $0.2\left|X_n\right|_{\max}$) series coefficients for $-0.75 \le t \le 0.75$. (**Hint**: Use a logical statement to find the non-negligible values, and find the appropriate value of N.)

3. One period of the periodic signal $x(t)$ is specified by

$$x_1(t) = \begin{cases} 0 & -2 \le t < -1.2 \\ 0.1 + 2\cos(2\pi t) & -1.2 \le t < 1.2 \\ 0 & 1.2 \le t < 2 \end{cases}$$

. Compute samples of $x_1(t)$ over the one period specified by the time interval $-2 \le t < 2$. Use a sample spacing of 0.002. Save these samples in row array **x1**. Also save the corresponding time value row array as variable **t1**.

a. Plot the signal $x(t)$ over the interval $-6 \le t < 6$ (three periods).

b. Plot the amplitude and phase spectra for $x(t)$ over the interval $-32 f_o \le f < 32 f_o$. These are the non-negligible spectrum samples. Use them to compute the signal power.

c. Plot the signals corresponding to signal bandwidths specified by significance factors 0.02, 0.1, and 0.2 over the same interval as Part a. Compute the power in each signal. What percent of the signal power from part b is contained in each of these signals?