Objectives:

To demonstrate double-sideband modulation signal and spectra characteristics. Also, to investigate the effect of local oscillator phase error in a synchronous demodulator.

Theory:

We use modulation to change a message signal's characteristics so that they match the characteristics of a transmission channel for effective and efficient transmission. For example, an audio signal cannot be efficiently radiated from a reasonable-length antenna for transmission in an atmospheric or space channel.

By modulating a carrier sinusoid having a frequency of $f_c$ Hz, we change the signal that we will transmit so that it has a spectrum near the frequency $f_c$ rather than a spectrum near zero. For example, in broadcast AM radio, a specific radio station might broadcast an audio signal having frequencies from 100 Hz to 5 kHz by using modulation that creates a signal having a bandwidth of 10 kHz centered at a frequency of 1 MHz. The receiver must be able to "demodulate" the signal. That is, it must be able to convert the transmitted signal back to the original message signal.

One method for performing the modulation is to use the message signal to change the amplitude of the carrier signal. This is referred to as amplitude modulation. In this experiment, we consider double-sideband suppressed-carrier, or simply double-sideband (DSB) modulation.

Double-Sideband Modulation

The block diagram for a simple DSB modulator is shown in Figure 10.1, where $m(t)$ is the message signal, $x_c(t)$ is the carrier signal, and $x_m(t)$ is the modulated signal

$$x_m(t) = A_m(t) \cos(2\pi f_c t)$$

$$x_c(t) = A_c \cos(2\pi f_c t)$$

Figure 10.1 Double-Sideband Modulator
Note that the amplitude, $A_c m(t)$, of the modulated sinusoidal signal is directly proportional to the message signal. Using the modulation theorem of Fourier transforms, we find that the spectrum of the modulated signal is

$$X_m(f) = \frac{A}{2} M(f - f_c) + \frac{A}{2} M(f + f_c)$$

(10.1)

where $M(f)$ is the message signal spectrum. Example spectra are shown in Figure 10.2.

![Diagram](image)

(a) Message Spectrum

![Diagram](image)

(b) Modulated Signal Spectrum

**Figure 10.2** Spectra for Double-Sideband Modulation

Note that the modulated signal spectrum is the message signal spectrum shifted up and down by the carrier frequency $f_c$ and multiplied by $A_c/2$. It contains a band of frequencies above the carrier frequency, called the upper sideband (USB), and a band of frequencies below the carrier
frequency, called the lower sideband (LSB), and no carrier term. Thus the modulation is called double-sideband suppressed-carrier (DSB).

The modulated signal can be demodulated in the receiver with the synchronous (or coherent) demodulator shown in Figure 10.3.

It is called a synchronous demodulator because the local oscillator (LO) in the receiver must produce a signal that is identical to the carrier, except for amplitude. If the LO has a time varying phase error $\theta(t)$ with respect to the carrier phase, then the demodulator output is

$$y(t) = A_r A_c L \frac{m(t)}{2} \cos(2\pi f_c t)$$

(10.2)

For a constant phase error, $\theta(t) = \phi$, the amplitude of the output is attenuated by the factor $\cos(\phi)$. A linearly varying phase error, $\theta(t) = 2\pi f_c t$, corresponds to a constant frequency error $f_c$. In this case, the output fades out and changes sign every $1/2f_c$ seconds. If a voice signal is being transmitted the sign change is not serious for low fade rates (small frequency error) since the human ear is not sensitive to the signal sign change. However, the fade is annoying. For larger frequency errors (approximately 10 Hz for voice signals), the output signal becomes garbled and no useful information is transmitted. Tighter specifications on frequency error are required for data signals.

The spectra for the signals in the demodulator with no LO phase error (using the modulation theorem and the ideal LPF) are shown in Figure 10.4.
Note that the output signal is the input signal multiplied by $\frac{A_A}{A_C}L/2$; therefore, the demodulator has recovered the message signal from the modulated signal.

**Laboratory Procedure:**

For all parts of this procedure, the message signal is $m(t) = 2\cos(40\pi t + 2.1) + 6\sin(80\pi t) + 4\cos(160\pi t - 0.9)$ and the carrier signal is $x_c(t) = 10\cos(800\pi t)$. Use the time interval $-0.5s \leq t \leq 0.4998s$ with a signal sample spacing of 0.0002s when computing data for these signals (Note: The strange upper limit on $t$ is used to produce a number of samples that can be factored into a large number of prime factors. This reduces computation time.)

For parts 1 and 2 below, plot all signals for the time interval $|t| \leq 0.04$ s and plot all amplitude spectra for the frequency interval $|f| \leq 1000Hz$.

1. Plot the carrier signal, the message signal, and the modulated signal for a DSB system. A vertical scale extending from -12 to 12 is convenient for the carrier and message signal plots. A vertical scale extending from -120 to 120 is convenient for the modulated signal. Plot the message signal and modulated signal amplitude spectra as three subplots in one figure. Convenient vertical sales are 0 to 4 for the message signal spectrum and 0 to 20 for the modulated signal spectrum.
2. For the DSB system, plot the synchronous demodulator internal signal \( y_1(t) \) and output signal \( y(t) \). Use an LO signal of \( 0.25 \cos(800 \pi t) \) and an ideal LPF gain of 0.8. Convenient vertical scales are -24 to 24 for \( y_1(t) \) and -12 to 12 for \( y(t) \). Plot the amplitude spectra for \( y_1(t) \) and \( y(t) \). A convenient vertical scale for both plots is from 0 to 4. Compare the output signal of the demodulator to the message signal. From a frequency domain point of view, what is the low-pass filter doing? Considering the time-domain plots of the internal signal \( y_1(t) \), and the output signal \( y(t) \), comment on the effects of the low-pass filter. Verify analytically, using the equation for \( y(t) \) shown in Figure 10.3, that \( y(t) = m(t) \) for this demodulator.

3. For part 3, plot all signals for the time interval \( |t| \leq 0.5s \). Repeat Part 2 using constant demodulator LO phase error of \( \phi = \pi/4 \) rad and \( \phi = \pi/2 \) constant LO frequency error of \( f = 1Hz \) and \( f_e = 5Hz \). Note the effects of the phase error and the two different frequency errors on the output signal.