# EXPERIMENT NUMBER 9
INDUCTOR CURRENT-VOLTAGE RELATIONSHIP

Purpose: To demonstrate the relationship between the voltage and current of a inductor.

Theory:

An inductor is a linear circuit element whose voltage and current are related by a differential equation, much like that of a capacitor. The behavior of a real capacitor is very close to the behavior of an ideal capacitor. However, a real inductor often behaves differently than an ideal inductor because of the effects of electrical losses. However, for most applications, an actual inductor behaves like an ideal inductor in series with a resistor.

For an inductor, the relationship between current and voltage [1] is:
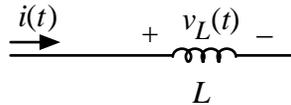
$$v_L(t) = L\frac{di}{dt}. \tag{1}$$

Actual inductors have a resistive term which accounts for the electrical losses,
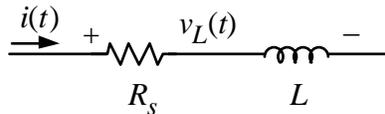
$$v_L(t) = i(t)R_s + L\frac{di}{dt}. \tag{2}$$

Unfortunately, it is not possible to observe both terms of (2). For a sinusoidal current at a particular frequency,

$$i(t) = I_m \sin(\omega t). \tag{3}$$

So,

$$v_L(t) = I_m R_s \sin(\omega t) + \omega L I_m \cos(\omega t). \tag{4}$$

If the voltage is written as

$$v_L(t) = V_m \sin(\omega t + \theta) \tag{5}$$

where θ is the phase difference between $v(t)$ and $i(t)$, and using the trigonometric identity,

$$\sin(x + y) = \sin x \cos y + \cos x \sin y, \tag{6}$$

equation (5) may be written as

$$v_L(t) = V_m \sin \omega t \cos \theta + V_m \cos \omega t \sin \theta \tag{7}$$

Equating terms of (4) and (7),

$$\begin{aligned} V_m \cos \theta &= I_m R_s \\ V_m \sin \theta &= \omega L I_m. \end{aligned} \tag{8}$$

Therefore, the L and R_s may be calculated,

$$\begin{aligned} R_s &= \left(\frac{V_m}{I_m}\right) \cos \theta \\ L &= \left(\frac{V_m}{\omega I_m}\right) \sin \theta \end{aligned} \tag{9}$$

where θ is the phase difference between $v_L(t)$ and $i(t)$, and ω is the radian frequency of the sinusoid, $\omega = 2\pi f$, where f is the frequency in Hz.

Experimental Procedure:
NOTE: Read through the calculations section prior to performing the procedure.
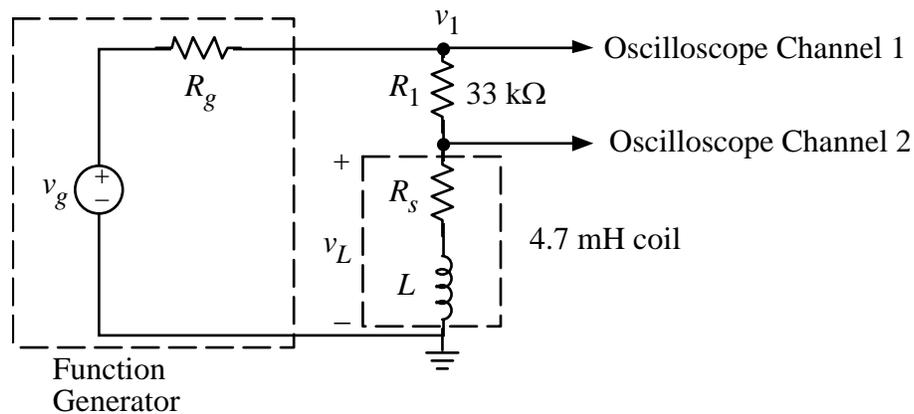   1. Assemble the following circuit with a prototype board:

Figure 2.

The large value of R_1 assures that, v_L, the voltage across the inductor, is small compared to v_1. Since v_L is small compared to v_1 the current, i, will be proportional to v_1.

2. Set the oscilloscope triggering to positive slope, and channel 1. Set the vertical mode to dual trace. Set the signal generator for a sinusoidal output of 1 kHz. Adjust the output level of the signal generator for 20 volts peak-to-peak, with a zero offset. Adjust the trigger level for a stable display. Change the volts/div for channel 2 so that the peak-to-peak amplitude of the trace for channel 2 is about 2 - 3 divisions. If a high frequency noise signal is superimposed on channel 2, press the "Scope BW" button to filter out most of this noise. Switch the input selector of each channel to ground and position the trace to coincide with the graticule at the center of the screen (zero volts and current). Switch the input coupling of both channels to DC. Adjust the time base of the oscilloscope to display 1 or 2 periods and accurately sketch the two waveforms. Record their period and peak values. As accurately as possible, obtain the phase angle difference between the two waveforms. The 10x horizontal magnification may be used to obtain an accurate reading.

3. Set the signal generator to produce a triangular signal so there is a 20 volt peak-to-peak signal output with zero offset. Accurately sketch the voltage and current waveforms. The observed waveform, v_L, is the sum of the resistive term, iR_s, and the inductive term, $L\frac{di}{dt}$.

4. Set the signal generator to produce a square wave so there is a 10 volt peak-to-peak signal with zero offset across the inductor. Accurately sketch the voltage and current waveforms.

5. Using an LCR meter, measure the apparent values of the inductance, L, and series resistance, R_s.

Calculations:
1. In part 2 of the experimental procedure, the current through the inductor was a sinusoidal signal.
    a) Did the current of the inductor lead or lag the voltage?

    b) Use the measured phase angle, θ, and the ratio of the magnitude of the inductor voltage and current, V_m/I_m, to determine the equivalent series resistance, R_s, and the inductance, L.

    c) Are the calculated R_s and L close to what was measured in part 5 of the experimental procedure? If not, why not?

2. In part 3 of the experimental procedure, the current through the inductor was a triangular waveform.

   a) Using the current and voltage waveforms, calculate R_s and L.

   b) Are the R_s and L close to what was measured in part 5 of the experimental procedure, or close to the values calculated in 1(b) above? Explain any discrepancies.

3. A parameter frequently used to characterize an inductor is its quality factor, Q, defined as

$$Q = \frac{\omega L}{R_S}$$

For a specified radian frequency, ω, Q is the ratio of the inductor reactance to its series resistance. A higher quality factor means the inductor resistance is small and the inductor is more nearly ideal. Calculate Q for the inductor.

References:

1   D.R. Cunningham and J.A. Stuller, *Circuit Analysis*, 2nd ed., Houghton Mifflin, Boston, 1995, p. 45.