For the function \( f(v, w, x, y, z) = m(1, 7, 8, 9, 10, 11, 13, 15, 17, 20, 22, 24, 31) \), answer the questions for parts a and b below.

a) Find the minimal Product of Sums (POS) expression for \( f \).

b) Implement the minimal POS expression from part a for \( f \) using the following logic gate combinations:
   1. AND Gates, OR Gates, Inverters with any number of inputs for these logic gates.
Part b continued)

2. NAND gates only with any number of inputs.

3. NOR gates only with any number of inputs.
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Simplify the expressions in a-d below using Boolean algebra only.

Note that y' denotes not y.

a) \[ G(w,x,y,z) = (w' + x + y + z') (w' + x + y + z') (w + x' + y' + z') (w + x' + y' + z) + wy'z \]

b) \[ F(a,b,c) = (a' + b'c')' + ab' \]
c) \( H(w,x,y,z) = wx'yz + wx + wy' + yz' \)

d) \( J(a,b,c,d) = ab'c'd + a'bc' + b' + cd \)
Design a binary mod-7 up/down counter. If the input $C = 0$, count up, and if $C = 1$, count down. Use "don't cares" for unused states and JK flip-flops for the state memory. Calculate next-state equations for all flip-flops.
Design a Mealy state machine that will detect a 1001 sequence with overlap (e.g. the following stream of bits will cause the sequence to be detected twice: "1001001"). When the sequence is detected, the output $Y$ will be High then reset itself to Low. Use SR flip-flops for state memory. Calculate next-state equations for each input ($S$ and $R$) of each flip-flop and an output equation for $Y$. 