A bridge is a special class of circuits that can be used for measuring resistance, capacitance, or inductance. A resistance bridge is especially useful when a very accurate measurement of a resistance is required. The Wheatstone bridge or four arm bridge, invented by C. Wheatstone in 1843, is the most widely used resistance bridge for measuring resistance values above 1 \( \Omega \). Commercial Wheatstone bridges are accurate to about 0.1 percent, making the values of resistance obtained far more accurate than values obtained from many types of meters. For resistances below 1 \( \Omega \), a Kelvin Bridge can be used [1].

A Wheatstone bridge consists of a voltage source and two parallel voltage dividers, as shown in Figure 1. The bridge is said to be balanced when \( v_{12} = 0 \). For the balanced condition, the voltage \( v_3 \) is divided in the path containing resistors \( R_a \) and \( R_b \) in the same ratio as in the path containing resistors \( R_c \) and \( R_x \), which allows the unknown resistance \( R_x \) to be determined in terms of \( R_a \), \( R_b \) and \( R_c \).

We can find \( R_x \) in terms of \( R_a \), \( R_b \) and \( R_c \) as follows. Using the voltage divider relation,

\[
v_1 = \frac{R_b}{R_b + R_a} v_3, \quad v_2 = \frac{R_x}{R_x + R_c} v_3
\]

For the balanced condition, \( v_{12} = 0 \), or \( v_1 = v_2 \). Equating the above expressions for \( v_1 \) and \( v_2 \) gives

\[
\frac{R_b}{R_b + R_a} = \frac{R_x}{R_x + R_c}
\]

Multiplying both sides by \((R_x + R_c)\) and \((R_b + R_a)\) gives \( R_b(R_x + R_c) = R_x(R_b + R_a) \). Subtracting \( R_b R_x \) from both sides and solving for \( R_x \) gives

\[
R_x = \frac{R_b R_c}{R_a}.
\]

In order to achieve balance for a specific unknown resistance \( R_x \), let \( R_a \) and \( R_c \) have fixed, known values, and let \( R_b \) be a calibrated (adjustable) resistor. The procedure is to adjust \( R_b \) until \( v_{12} = 0 \), and then use the expression derived above to determine \( R_x \).
Lab Work

1. Construct the Wheatstone bridge shown in Figure 1. Use resistor values $R_a = 1 \, \text{k}\Omega$, $R_c = 10 \, \text{k}\Omega$, and $R_s = 10 \, \text{k}\Omega$. Use a decade resistance box for $R_b$ and a DC power supply adjusted to 5 volts.

2. Measure the value of an "unknown" resistance supplied by your lab GTA $1 \, \text{k}\Omega \leq R_x \leq 10 \, \text{k}\Omega$. In adjusting $R_b$ using the decade resistance box, start with the coarsest scale and work toward the finest scale, while monitoring $v_{12}$ with the DMM.

3. Calculate the unknown resistance.

Next, perform one (or both, if time permits) of the following exercises (your choice):

4. Measure the resistance of a photo-resistor under various lighting conditions.

5. Measure the resistance of ten different $1 \, \text{k}\Omega$ resistors and plot their values in terms of percent deviation from the nominal value. A bar graph is a convenient form for this plot.