EE 2101 - EXPERIMENT 3
RESISTIVE NETWORKS AND COMPUTATIONAL ANALYSIS

INTRODUCTION

The resistors used in this laboratory are carbon composition resistors, consisting of graphite or some other type of carbon embedded in a filler material. Graphite is a moderately good conductor, so by varying the graphite-filler mix, a large range of resistance values can be obtained (the less graphite, the higher the resistance) [1]. Carbon resistors are cheap and reliable, however, their tolerances (5 to 20% deviation from nominal values) indicate that larger errors can be expected. Other types of resistors include wire wound, metal film, and carbon film [1].

The nominal value and tolerance of a carbon resistor can be determined from the color-coded stripes that appear on the resistor as shown in Table 1 and Figure 1. The first two bands represent the two significant figures of the resistance, while the third band indicated the number of zeros that follow. If there are only three bands, the resistor has a 20% tolerance. If the three color bands are followed by a silver band, the resistor has a 10% tolerance. A gold band following the three color bands indicates a 5% tolerance, a red band indicates a 2% tolerance, and a brown band indicates a 1% tolerance. For example, a resistor with bands of yellow, violet, red, and gold has a resistance of 4700 Ω ±5%. *Therefore, the measured value of the resistance should be between 4465 Ω and 4935 Ω.*

Table 1. Resistor Four-Band Color Code

First Three Color Bands (In Order: First Significant Digit, Second Significant Digit, and Zeros/Multiplier)					
Band Color	Black	Brown	Red	Orange	Yellow
Value	0	1	2	3	4
Band Color	Green	Blue	Violet	Gray	White
Value	5	6	7	8	9
Last Color Band (Tolerance)					
Band Color	None	Silver	Gold	Red	Brown
Value	20%	10%	5%	2%	1%

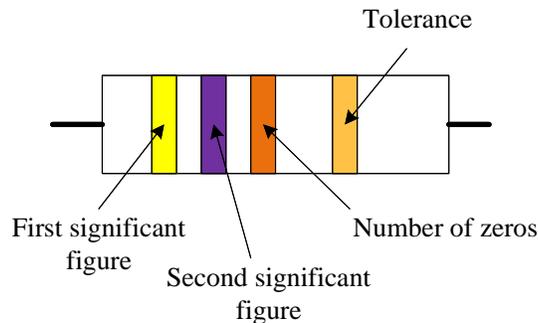First significant figure

Second significant figure

Number of zeros

Tolerance

Figure 1. Example Resistor of Nominal Value 47 kΩ ±5%.

The power rating of a resistor is the maximum power the resistor can handle. If this power is exceeded,

the resulting current flowing in the resistor will transfer too much energy (in the form of heat) to the resistor material, causing damage to the resistor. A carbon composition resistor could warp, create an open circuit, and possibly explode when the power rating is significantly exceeded. Resistors should be visually inspected for existing warping or other damage, and resistance values should be measured with the DMM or LCR meter prior to use. The maximum power that a carbon composition resistor can handle is indicated by its size. Standard values are 1/8, 1/4, 1, and 2 watt ratings. A 2-watt resistor is over a centimeter in diameter, while a 1/8-watt resistor is a couple of millimeters in diameter.

To determine the power rating of a resistor for use in a circuit, estimate the maximum voltage or current that will pass through the resistor, and then compute the power from $P = \frac{V^2}{R} = I^2 R$. To be on the safe side, it can be assumed that the calculated value might possibly be exceeded by a factor of 4, and then use a resistor with an appropriate power rating.

Statistical Analysis
In this laboratory you will be collecting data and you will be analyzing a large data set. It is often useful to apply simple descriptive statistics to analyze your data. You might be calculating how much error you have in a measurement or analyzing data collected from a sample of items. Table 2 and 3 lists the statistical measures that you will be using in this laboratory.

Table 2. Descriptive Statistical Parameters: Measures of Center

Measure	**Definition and Calculation**
Mean	Arithmetic average of a set of n numbers
	$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$
Median	The middle data point in an ordered list of n numbers
	middle value if n is odd *average of two middle values if n is even*

One of the applications of descriptive statistics is to find the center of your data. The two measures of center you will use are the mean (or arithmetic average) and the median (or the middle data point in an ordered list) of the sample. The symbol for a sample mean is $\bar{x}$. The n in the following formulas is the number of data points in your sample.

Table 3. Descriptive Statistical Parameters: Measures of Variability

Measure	Definition and Calculation
Maximum	max( ) is the largest data value in the set
Minimum	min( ) is the smallest data value in the set
Range	maximum data value minus minimum data value
Standard Deviation	a measure of the variability (spread) of a data set containing n data points and is based up on the deviation of each data point from the mean $$s = \sqrt{\dfrac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n-1}}$$
Quartiles Q1 and Q3	Q1=first quartile=lower fourth of the data $$Q_1 = \begin{cases} \textit{median of smallest n/2 observations if n is even} \\ \textit{median of the smallest (n+1)/2 observations if n is odd} \end{cases}$$ Q3=third quartile=upper fourth of the data $$Q_3 = \begin{cases} \textit{median of largest n/2 observations if n is even} \\ \textit{median of the largest (n+1)/2 observations if n is odd} \end{cases}$$

Another common application of descriptive statistics is to find the spread or variability in your data. There are many measurements that can be applied. In the first part of this laboratory you will calculate the range and the standard deviation. The range is the simplest measure of variability in a data set but it also the least useful for further analysis. The sample standard deviation is a more useful measure of variability. It is based upon the deviation of each data point from the mean. The larger the standard deviation, the more spread out your data value are. However, a single data value that lies very far from the mean (an outlier) can drastically affect the standard deviation. Other measures of spread which are not so sensitive to outliers will be discussed in future labs.

When taking measurements in the laboratory, you should calculate the percent error in your measurement. This is a different type of variability since it applies to a single reading and not a set of data, but it is very useful. You should compare your measurement to the theoretical or nominal value and then seek to find an explanation for any large errors.

$$\% \text{error} = \frac{\textit{Nominal - Measured}}{\textit{Nominal}} \text{x}100$$

It is often useful to graph your data so that you might be able to detect trends or patterns in the data. In this lab you will plot a dot diagram. A dot diagram is a simple plotting of data along a single axis or number line. Using dots, plot each of the data values in your sample and then look for patterns. Dot diagrams and histograms are useful for small data sets. Alternatively, the data can be graphed in a boxplot as given in Figure 2. In the boxplot, the median and quartiles are shown explicitly on a line segment. Outliers are data values that lie outside the interval $\left(Q_1 - 1.5IQR, Q_3 + 1.5IQR\right)$. If they exist, they are plotted as dots.
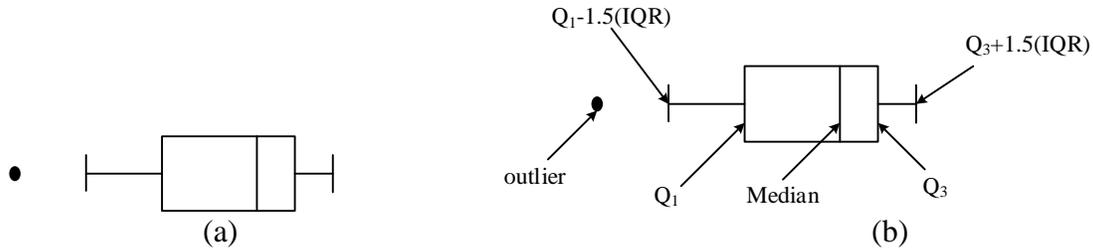
Figure 2. a) Boxplot and b) Boxplot with Key Measures Identified.

This laboratory has two parts. In Part A you work with experimental resistor networks. In Part B you work with a randomly generated resistance set. You will do statistical analysis for both parts.

OBJECTIVES

1. Introduce the Resistor Color Code.
2. Investigate Kirchhoff's current and voltage laws for resistor networks.
3. Introduce statistical analysis related to resistor tolerances.
4. Introduce circuit simulation using computer tools.
5. Investigate statistical analysis related to resistor tolerances.

EXPERIMENTAL PROCEDURE FOR PART A

In this laboratory you will construct resistance networks and perform voltage and current measurements to verify some of the basic network relationships you have learned, including Kirchoff's current law (KCL), Kirchoff's voltage law (KVL), the current divider relation, and the voltage divider relation. In performing the measurements, keep in mind the internal meter resistance for the DMM determined in the last lab. Note any instances where this resistance is a significant source of error.

Task A1

Collect 1/4-watt resistors with values of $R_1 = 4.7$ kΩ, $R_2 = 3.3$ kΩ, $R_3 = 10$ kΩ, $R_4 = 1$ kΩ, and $R_5 = 10$ kΩ. Using the ohmmeter feature of the digital multimeter (DMM), determine the actual values of the resistances. Record the measured resistance values and the tolerance for each resistor. If any of the measured values are not close to the nominal values, double check the resistor labeling and replace the resistor.

Task A2

Using a breadboard, assemble the network shown in Figure 3. Using the DMM as a voltmeter, adjust the supply voltage, V, from the DC power supply, to obtain ten volts.
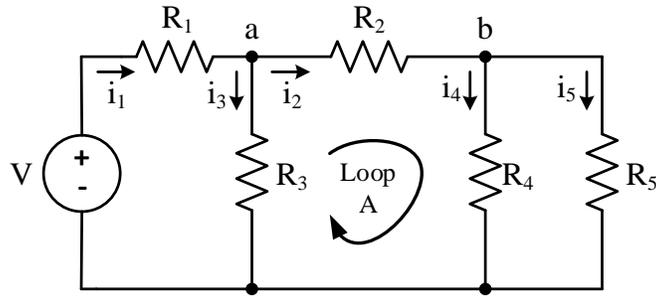
Figure 3. Resistor Network with Nodes a and b, note Loop A.

Task A3

Measure and record the voltage across each resistor shown in Figure 3. Be sure to record the polarities of the voltages.

Task A4

Using the DMM as an ammeter, measure and record the currents i_1 though i_5 shown in Figure 3. You will have to insert the ammeter in series with the resistors one-by-one, and then replace the resistor to its original position prior to doing the next measurement. Keep track of the direction of the currents you measure.

Task A5

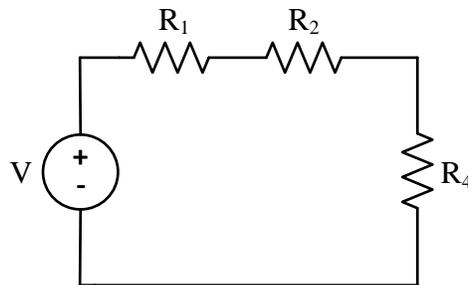Remove resistors R3 and R5 from your circuit to create the circuit shown in Figure 4.

Figure 4. Voltage Divider with Three Series Resistors.

Task A6

Measure and record the voltages across the three resistors shown in Figure 4. Be sure to record the polarities of the voltages.

Task A7

Collect ten resistors with the same nominal value. Measure the resistance value of each with the LCR meter. Record the measured value and the tolerance (indicated by the color-coded band) of each resistor. Keep the resistors sorted so that you know which one has what measured value.

Task A8

Using a breadboard, assemble the network shown in Figure 5. Using the DMM as a voltmeter, adjust the supply voltage, V, from the DC power supply, to obtain ten volts. For R_1 use the resistor from step 7 which has the smallest measured value. For R_2 use the resistor from step 7 which has the largest measured

value. Measure and record the voltages across each of the resistors. Be sure to record the polarities of each of the voltages.
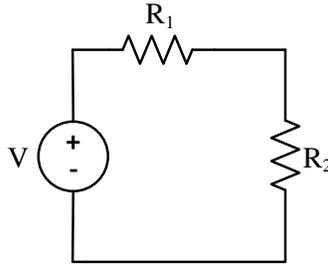

Figure 5. Voltage Divider with Two Series Resistors.

Analysis for Part A
To written in the laboratory notebook.

1. Verify KVL for the center loop (Loop A) as shown in Figure 3.

2. Verify KCL for node a as shown in Figure 3.

3. Verify the current divider relation for the division of current i_2 as it passes through resistors R_4 and R_5. You can do this using the currents you measured in procedure step 4.

4. Verify the voltage divider relation for the circuit shown in Figure 4.

5. Calculate the percent error from the nominal value for each of the resistors measured in procedure step 7. Do any of the resistance values lie outside the allowed tolerance range based up on the color-coded band on the resistor?

6. Make a dot diagram for the set of resistor values found in procedure step 7. The endpoints of your line segment should be (nominal value ± tolerance). Then plot the values of each of the 10 resistors on the line segment. Do you see a pattern in the data? If so, what might be causing such this pattern?

7. Calculate the mean (arithmetic average) $\bar{x}$, and median of the resistance values for the ten resistors measured in step 7. Are these values close to the nominal value?

8. Calculate the sample standard deviation, s, for the resistance values measured in procedure step 7 where n is the number of resistors in the sample (in this case, n=10). The x_i's in the equation are the individual resistor values. Calculate the maximum value, minimum value, and range. How do the maximum and minimum values compare to the endpoints of the dot diagram?

9. If both resistors were exactly the nominal value, calculate what the voltage would be across each of the resistors in the circuit in Figure 5.

10. Calculate the percent error between the measured voltage value found in procedure step 8 and the "nominal" value found in the previous analysis step.

11. Calculate the resistor tolerance level required if the output of the voltage divider can vary no more than 5% from the "nominal" value found in analysis step 9.

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Experimental Procedure: Part B
In this lab, a randomly generated resistance set will be analyzed to understand the variations in the resistances and other statistically important parameters like minimum value, maximum value, mean, standard deviation, etc.

Task B1
Use Excel to generate a set of 100 randomly generated resistance values. Your instructor will assign you one of the following resistor values to create the set – 100 Ω, 330 Ω, 470 Ω, 1000 Ω, 3300 Ω, or 4700 Ω. Your instructor will also assign one of the following tolerance values - 1%, 5%, or 10%. Create a new excel sheet and save it as "Last name_first name_Circuits1_Labsection".

Use the command " = **RANDBETWEEN( )**" in a blank cell. In the parenthesis, mention the bottom and top values as calculated for your chosen resistance and tolerance values. For example, if you chose 100 Ω and a tolerance of 5%, the bottom value will be (100 – 5%) Ω and the top value will be (100 + 5%) Ω. This should generate one random value. Hold the bottom right corner of the cell in which the value was generated and pull it down the column to generate 100 random values.

The following instructions are provided assuming that the data set is located in column A and the data goes from Cell 1 to Cell 100. Choose a different cell for each command and keep all the previously calculated values. Select the set of randomly generated number, copy those values and then choose the top cell of column C to paste them. Click on the paste (clipboard) icon and choose "paste special". Select values and paste the values in the chosen column. This will ensure that you are using the same set of values. Otherwise you may notice that the values keep changing to generate new random numbers.

Task B2
Find the mean of the randomly generated resistances. Choose an empty cell and use the command " = **average(C1:C100)**" assuming you pasted you values in column C. Choose a cell right next to the cell you are calculating a value in to label the value, e.g.

999.42	Mean

Task B3
Find the sample median. Use the command " = **median(C1:C100)**" in an empty cell. You should see a value that numerically lies right in the middle of the set. You can also use the "sort" icon to sort the data values in ascending or descending order thus verifying the median. Sorting the data will also help you verify or correlate some of the parameters that are to be found in the following tasks.

Task B4
Find the maximum value and minimum value amongst the randomly generated set. Use the command " = **max(C1:C100)**" to display the maximum resistance in the set. Use the command " = **min(C1:C100)**" to display the minimum resistance in the set.

Task B5
Find the standard deviation. Use the command " = **stdev(C1:C100)**" to display the standard deviation in the set.

Task B6
Find the Quartiles Q1 and Q3, i.e. the 25th percentile and the 75th percentile of the sorted data, respectively.   This can be found by using the command **" = percentile(c1:c100, 0.25)"** and

" = **percentile(c1:c100, 0.75**)", respectively.

Task B7

Obtain the box plot of the sorted data according to Quartiles Q1 and Q3. Plot Q1, median and Q3 using "clustered stacked bars" under the "charts" tab. By right clicking the chart, choose "format data series". Then under "options", select the options to show different colors for the three data points and set the series overlap 100%.

Arrange the three stacked bars such that they are overlapping choosing the order from bottom to top as 75th percentile, median and 25th percentile. Make the 25th percentile invisible by making the fill color to white.

Add "error bars" to Q1 and Q3 series by choosing "error by percentage". Error bars feature can be found under the "chart layout" tab. You can choose the percentage to show a smaller or larger range of outliers. Note the other error-bar options.

Analysis for Part B
To written in the laboratory notebook. Print your Excel file data for the report.

1. Note your assigned nominal resistance and tolerance. Prepare a table showing the following for the data set: mean, median, maximum value, minimum value, range, standard deviation, etc.

2. From Step 7, show the boxplot. Prepare a table showing the median and the Q1 and Q3 quartile values.

Error bars could be used to display the expected variation of values around the mean. Error-bar designation could be defined using the maximum and minimum values, the standard deviation, the quartile values, or a given percentage. Describe how many outliers occur in each of these four definitions. Discuss how the number of outliers would change if the data distribution was not uniform across the range, but followed a normal distribution (also called a bell curve or Gaussian distribution).

[1] Wolf, Stanley, *Guide to Electronic Measurements and Laboratory Practice*. Prentice-Hall, Englewood Cliffs, New Jersey, 1973.